

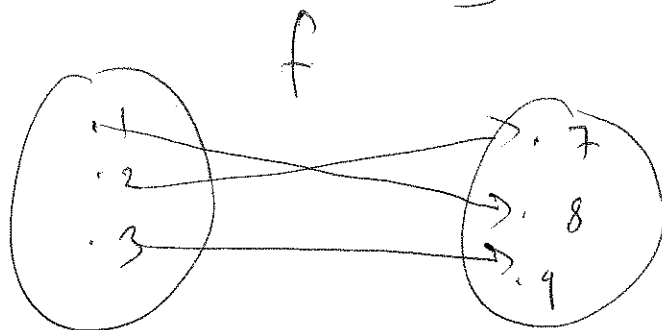
2.1: Properties of functions

A function is a rule that assigns to each element in some set D , exactly one element, $f(x)$, to a set B .

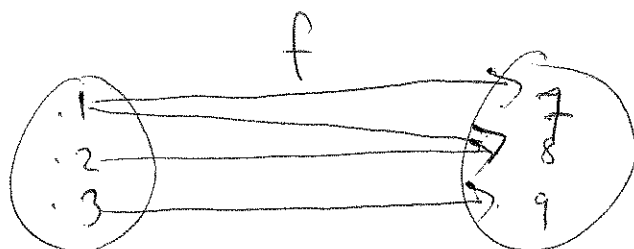
The domain D is the set of all possible inputs.

The range R is the set of all corresponding outputs.

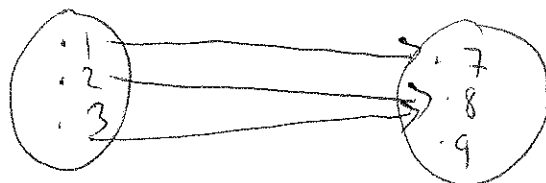
Determine if the following representations of f is a function



Yes, f is a function



no since $f(1) = 7$ and
 $f(1) = 8$



Yes

Given

x	-1	0	1	2	-1
y	2	4	5	7	3

a) Is y a function of x?
no

b) Is x a function of y?
Yes.

Is y a function of x?

given $y^2 = x+1$

no, since $y = \pm \sqrt{x+1}$

For example $x=3$ gives

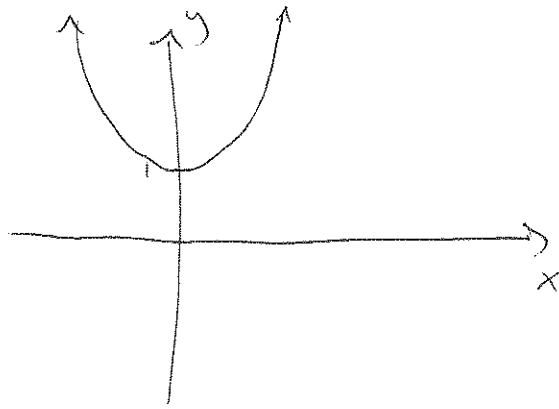
$y=2$ and $y=-2$

Example. For the following functions, find the domain and the range:

1. $f(x) = x^2 + 1$

$D = (-\infty, \infty)$

$R = [1, \infty)$



2. $g(x) = \sqrt{2x - 4}$

$2x - 4 \geq 0$

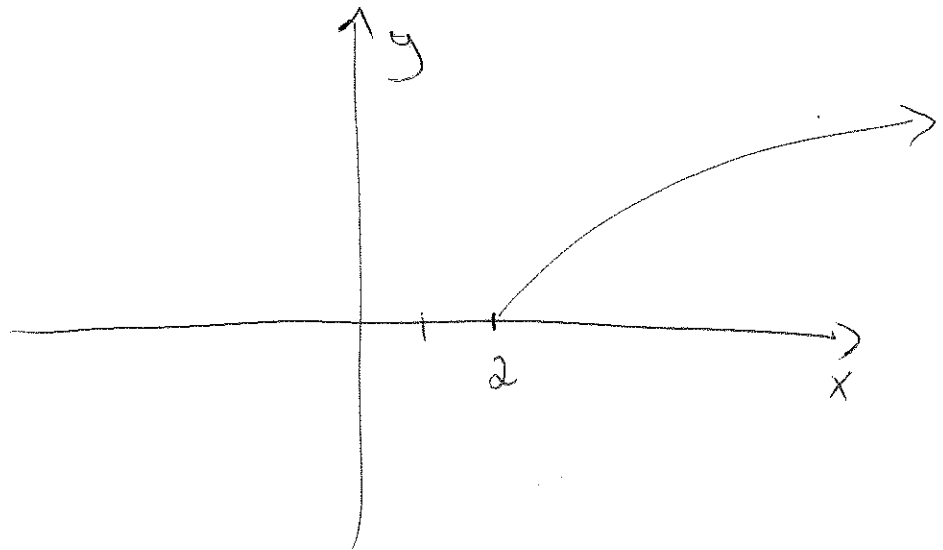
$2x \geq 4$

$x \geq 2$

$D = [2, \infty)$

$= \{x : x \geq 2\}$

$R = [0, \infty)$



3. $h(x) = \frac{1}{x^2 - 25}$

$h(x) = \frac{1}{(x+5)(x-5)}$

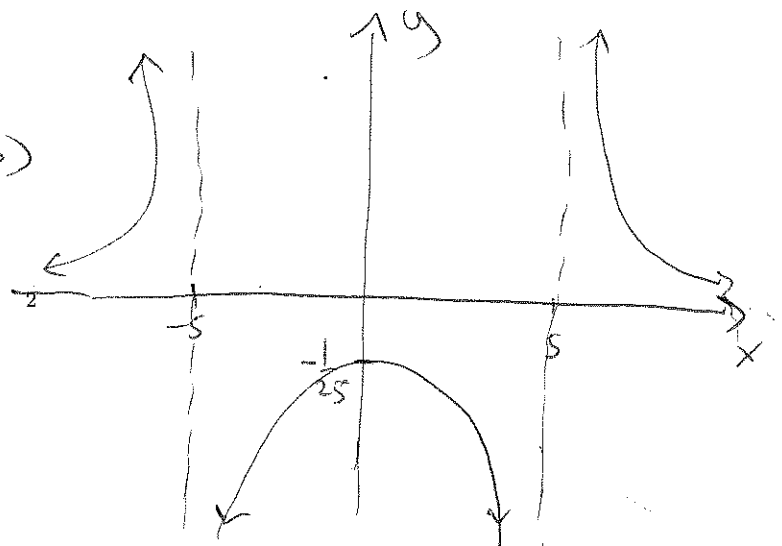
$x+5=0 \Rightarrow x=-5$

$x-5=0 \Rightarrow x=5$

$D = \{x : x \neq \pm 5\}$

$= (-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

$R = (-\infty, 0) \cup (0, \infty)$



Example. Given

$$f(x) = x^2 - 5.$$

Find:

$$1. f(x+h) = (x+h)^2 - 5 = x^2 + 2xh + h^2 - 5$$

$$2. f(x+h) - f(x) = x^2 + 2xh + h^2 - 5 - (x^2 - 5) \\ = 2xh + h^2$$

$$3. \frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2}{h} = 2x + h$$

Definition. The vertical line test: A graph represents a function if and only if every vertical line intersects the graph in at most one point.

function?

