1.2: Linear functions and applications

For $y = 2x + 5$, we say that y is a function of x
Recall, that $f(x)$ is read "f of x", and we say that f is a function of x.

Definition. A linear function is a function on the form,

$$y = f(x) = mx + b,$$

where m is the slope of the line and b is the y-intercet. We call x the independent variable and y the dependent variable.

Example. Let $f(x) = 4x - 3$. Find the following:

1. $f(2)$ = $4 \cdot 2 - 3 = 5$

2. $f(-3)$ = $4 \cdot (-3) - 3 = -15$

3. $f(h+2)$ = $4(h+2) - 3 = 4h + 8 - 3 = 4h + 5$

The demand curve for a given product relates the quantity q of the product demanded by the consumers during some time period to the price p per unit of the product. As the price increases, the smaller quantity is demanded. We plot p as a function of q.
The supply curve for a given product relates the quantity q of the product that producers are willing to make during some time period to the price p per unit of the product. Usually, if the price increases, so will the quantity supplied. Again, we plot p as a function of q.

The quantity at which the demand $p = D(q)$ is equal to the supply, $p = S(q)$, i.e $D(q) = S(q)$ is called the **equilibrium quantity** and the corresponding price is called the **equilibrium price**.

Example. Suppose the demand curve for the demands of apples is given by

$$p = D(q) = -0.005q + 10,$$

where q is the quantity demanded in thousands of boxes of apples per week and p is the price in dollars. Suppose the supply curve for the supply of apples is given by

$$p = S(q) = 0.01q + 5.8,$$

where q is the quantity supplied in thousands of boxes of apples per week and p is the price in dollars.

1. Find the quantity demanded at a price of \$9 per box.

$-0.005q + 10 = q$

$-0.005q = -1$

$q = 200.$

The quantity demanded is 200,000 boxes

2. Find the quantity supplied at a price of \$9 per box.

$0.01q + 5.8 = q$

$0.01q = 3.2$

$q = 320$

The quantity supplied is 320,000 boxes.

3. Find the equilibrium quantity.

$-0.005q + 10 = 0.01q + 5.8$

$0.015q = 4.2$

$q = 280$

280,000 is the equilibrium quantity.

4. Find the equilibrium price.

$S(280) = 0.01 \cdot 280 + 5.8 = 8.6$

The equilibrium price is \$ 8.6 per box

The cost function $C(q) = mq + b$ gives the cost of producing a quantity q of some product. The slope m represents the marginal cost and the y-intercept represents the fixed cost. The marginal cost is the cost of producing one additional item.

The revenue function $R(q)$ gives the total revenue from selling q units of an item at a price of p per unit. It is given by

$$R(q) = pq.$$

The profit function $P(q)$ is given by $P(q) = R(q) - C(q)$, the difference between revenue $R(q)$ and cost $C(q)$. The company makes profit if $P(q) > 0$. The quantity q for which $R(q) = C(q)$ is called the **break-even quantity**.

Example. Suppose a company found that the fixed cost of producing a product is \$50 and that the cost of producing 100 items of the product is \$70. Assume the cost $C(q)$ is a linear function of q, the quantity produced. They sell the product for \$2.2 per item.

1. Find a formula for $C(q)$

$C(q) = m \cdot q + 50$ $\quad$ $(0, 50)$ and $(100, 70)$ are points on the curve.

$m = \frac{70-50}{100-0} = \frac{20}{100} = 0.2$

$C(q) = 0.2q + 50$

2. What is the marginal cost? The marginal cost is $m = 0.2$ dollar

3. Find a formual for the revenue, $R(q)$.

$R(q) = 2.2 \cdot q$

4. How many items must be sold for the company to break even?

$C(q) = R(q)$

$0.2q + 50 = 2.2q$

$2q = 50$

$q = 25$

The company must sell 25 items to break even

5. What is the profit if 70 items of the product is sold?

$P(q) = R(q) - C(q) = 2.2q - (0.2q + 50) = 2q - 50$

$P(70) = 2 \cdot 70 - 50 = 90$

The profit is \$ 90