1.2: Linear functions and applications

For $y = 2x + 5$, we say that y is a function of x
Recall, that $f(x)$ is read "f of x", and we say that f is a function of x.

Definition. A linear function is a function on the form,

$$y = f(x) = mx + b,$$

where m is the slope of the line and b is the y-intercet. We call x the independent variable and y the dependent variable.

Example. Let $f(x) = 4x - 3$. Find the following:

1. $f(2)$

2. $f(-3)$

3. $f(h + 2)$

The demand curve for a given product relates the quantity q of the product demanded by the consumers during some time period to the price p per unit of the product. As the price increases, the smaller quantity is demanded. We plot p as a function of q.
The supply curve for a given product relates the quantity q of the product that producers are willing to make during some time period to the price p per unit of the product. Usually, if the price increases, so will the quantity supplied. Again, we plot p as a function of q.

The quantity at which the demand $p = D(q)$ is equal to the supply, $p = S(q)$, i.e $D(q) = S(q)$ is called the **equilibrium quantity** and the corresponding price is called the **equilibrium price**.

Example. Suppose the quantity q demanded in thousands of boxes of apples per week and the price p (in dollars) per box is related by

$$p = D(q) = -0.005q + 10.$$

Suppose the quantity q supplied in thousands of boxes of apples per week and the price p (in dollars) per box is related by

$$p = S(q) = 0.01q + 5.8.$$

1. Find the quantity demanded at a price of \$9 per box.

2. Find the quantity supplied at a price of \$9 per box.

3. Find the equilibrium quantity.

4. Find the equilibrium price.

The cost function $C(q) = mq + b$ gives the cost of producing a quantity q of some product. The slope m represents the marginal cost and the y-intercept represents the fixed cost. The marginal cost is the cost of producing one additional item.

The revenue function $R(q)$ gives the total revenue from selling q units of an item at a price of p per unit. It is given by

$$R(q) = pq.$$

The profit function $P(q)$ is given by $P(q) = R(q) - C(q)$, the difference between revenue $R(q)$ and cost $C(q)$. The company makes profit if $P(q) > 0$. The quantity q for which $R(q) = C(q)$ is called the **break-even quantity**.

Example. Suppose a company found that the fixed cost of producing a product is \$50 and that the cost of producing 100 items of the product is \$70. Assume the cost $C(q)$ is a linear function of q, the quantity produced. They sell the product for \$2.2 per item.

1. Find a formula for $C(q)$

2. What is the marginal cost?

3. Find a formual for the revenue, $R(q)$.

4. How many items must be sold for the company to break even?

5. What is the profit if 70 items of the product is sold?