Practice TEST #4 - Answer Key

(1) Express in terms of sums and differences of logarithms:

(a) $\log_a(6xy^5z^4) = \log_a(6) + \log_a(x) + 5\log_a(y) + 4\log_a(z)$

(b) $\log_a\left(\sqrt{\dfrac{a^6b^8}{a^2b^5}}\right) = 2 + \dfrac{3}{2}\log_a(b)$

(c) $\log_a\left(\sqrt[4]{\dfrac{m^8n^{12}}{a^3b^5}}\right) = 2\log_a(m) + 3\log_a(n) - 3 - \dfrac{5}{4}\log_a(b)$

(2) Express as a single logarithm and, if possible simplify:

(a) $\dfrac{1}{2}\log_a x + 4\log_a y - 3\log_a x = \log_a\left(\dfrac{y^4}{x^{\frac{5}{2}}}\right)$

(b) $\ln 2x + 3(\ln x - \ln y) = \ln\left(\dfrac{2x^4}{y^3}\right)$

(c) $\dfrac{2}{3}[\ln(x^2-9) - \ln(x+3)] + \ln(x+y) = \ln(x-3)^{\frac{2}{3}}(x+y)$

(3) Find the value of

$$x = \frac{\log 3^4 - \log 2}{\log 2^3 + \log 3} = 1.164644$$

(4) Given that $\log_a x = 2$, $\log_a y = 3$, and $\log_a z = 4$ find

$$\log_a \frac{\sqrt[4]{y^4z^5}}{\sqrt[4]{x^3z^{-2}}} = \frac{17}{2}$$

(5) Solve:

(a) $2^{3x-7} = 32$, $x = 4$

(b) $3^x = 20$, $x = \dfrac{\ln 20}{\ln 3}$

(c) $e^{0.08t} = 2500$, $t = \dfrac{\ln 2500}{0.08}$

(d)$27 = 3^{5x} \cdot 9^{x^2}$, $x = \frac{1}{2}$, $x = -3$

(e)$5^{x+2} = 4^{1-x}$, $x = \frac{\ln 4 - 2\ln 5}{\ln 5 + \ln 4}$

(f)$1000e^{0.09t} = 5000$, $t = \frac{\ln 5}{0.09}$

(6) Solve:

(a) $\log_3 x = -2$, $x = \frac{1}{9}$

(b) $\log x + \log(x+3) = 1$, $x = 2$

(c) $\log_3(2x-1) - \log_3(x-4) = 2$, $x = 5$

(d)$\log_2(10+3x) = 5$, $x = \frac{22}{3}$

(e)$\ln(x+1) - \ln x = \ln 4$, $x = \frac{1}{3}$

(f)$\log(2x+1) - \log(x-2) = 1$, $x = \frac{19}{8}$

(7) In 1626, Peter Minuit of the Dutch West India Company purchased Manhattan Island from the Indians for \$24. Assuming an exponential rate of inflation of 8% per year, how much will Manhattan be worth in 2005?

(8) Karen Guardino, who is self-employed, wants to invest \$60,000 in a pension plan. One investment offers 7% compounded quarterly. Another offers 6.75%, compounded continuously.

(a) Which investment will earn more interest in 5 years? How much more will the better plan earn?

(b) If Karen chooses the plan with continuous compounding, how long will it take for her \$60,000 to grow to \$80,000?

(9) The radioactive element carbon-14 has a half-life of 5750 yr. The percentage of carbon-14 present in the remains of organic matter can be used to determine the age of that organic matter. Archaeologists discovered that the linen wrapping from one of the Dead Sea Scrolls had lost 22.3% of its carbon-14 at the time it was found. How old was the linen wrapping?

(10) One action that government could take to reduce carbon emission into the atmosphere is to place tax on fossil fuel. This tax would be based on the amount

of carbon dioxide emitted into the air when the fuel is burned. The cost-benefit equation

$$\ln(1 - P) = -.0034 - .0053T$$

describes the approximate relationship between a tax of T dollars per ton of carbon and the corresponding percent reduction P (in decimal form) of emission of carbon dioxide.

(a) Write P as a function of T.
(b) Graph P for $0 \le T \le 1000$.
(c) Determine P when $T = \$60$, and interpret this result.
(d) What value of T will give a 50% reduction in carbon emission?

(11) Match the logarithm in Column I with its value in Column II.

I			II
1. $\log_2 16$	--→	E	A. 0
2. $\log_3 1$	--→	A	B. $\frac{1}{2}$
3. $\log .1$	--→	G	C. 5
4. $\log_2 \sqrt{2}$	--→	B	D. not a real number
5. $\log 10^5$	--→	C	E. 4
6. $\ln\left(\frac{1}{e^2}\right)$	--→	H	F. -3
7. $\log_{1/2} 8$	--→	F	G. -1
8. $\log_5(-1)$	--→	D	H. -2

(12) For each statement, write an equivalent statement in logarithmic form.
(a) $2^5 = 32$, $\log_2(32) = 5$

(b) $10^{-4} = .0001$, $\log(.0001) = -4$

(13) For each statement, write an equivalent statement in exponential form.
(a) $\log_6 36 = 2$, $6^2 = 36$

(b) $\log_{\sqrt{3}} 81 = 8$, $(\sqrt{3})^8 = 81$

(14) Sketch the graph of $f(x) = \log_2 x$. Using transformations of it, graph each function.
(a) $f(x) = \log_2(x + 3)$

(b) $f(x) = -\log_2 x - 3$

(15) Write each expression as a sum, difference, or product of logarithms. Simplify the result if possible.

(a) $\log_5\left(\frac{5\sqrt{7}}{3}\right) = 1 + \frac{1}{2}\log_5(7) - \log_5(3)$

(b) $\log_4(2x+5y)$

(c) $\log_p \sqrt[3]{\frac{m^5n^4}{t^2}} = \frac{5}{3}\log_p(m) + \frac{4}{3}\log_p(n) - \frac{2}{3}\log_p(t)$

(16) Write each expression as a single logarithm with coefficient 1.

(a) $(\log_b k - \log_b m) - \log_b a = \log_b\left(\frac{k}{ma}\right)$

(b) $\frac{1}{2}\log_y(p^3q^4) - \frac{2}{3}\log_y(p^4q^3) = \log_y\left(\frac{1}{\sqrt{p}}\right)$

(c) $2\log_a(z-1) + \log_a(3z+2) = \log_a(z-1)^2(3z+2),\ z > 1$

(17) Solve

$$\ln e^{\ln e} - \ln(x-3) = \ln 2$$

, $x = \frac{e+6}{2}$

(18) How long will it take an investment to triple, if interest is compounded continuously at 5%?

(19) A midwestern city finds its residents moving to the suburbs. Its population is declining according to the relationship

$$P = P_0e^{-.04t},$$

where t is the time measured in years and P_0 is the population at time $t = 0$. Assume that $P_0 = 1,000,000$.

(a) Find the population at time $t = 1$.

(b) Estimate the time it will take for the population to decline to 750,000.

(c) How long will it take for the population to decline to half the initial number?

(20) Write the first five terms of each sequence.

(a) $a_n = 4n + 10$

$$14, 18, 22, 26, 30$$

(b) $a_n = (-1)^{n-1}(n+1)$

$$2, -3, 4, -5, 6$$

(c) $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$, for $n \geq 3$

$$1, 1, 2, 3, 5$$

(21) Write the first five terms of the arithmetic sequence with the first term 8, and the common difference is 6.

$$8, 14, 20, 26, 32$$

(22) Find a_8 and a_n for each arithmetic sequence.

(a) $a_1 = -3, d = -4$

$a_n = -3 + (n-1)(-4)$, $a_8 = -31$

(b) $a_5 = 27, a_{15} = 87$

$a_n = 3 + (n-1)6$, $a_8 = 45$

(c) $a_1 = -4, a_5 = 16$

$a_n = -4 + (n-1)5$, $a_8 = 31$

(d) $a_3 = 8, d = -3$

$a_n = 14 + (n-1)(-3)$, $a_8 = -7$

(23) Find a_8 and a_n for each geometric sequence.

(a) $a_1 = 5, r = -2$

$a_n = 5(-2)^{n-1}$, $a_8 = 5(-2)^7$

(b) $a_2 = -6, a_7 = -192$

$a_n = (-3)2^{n-1}$, $a_8 = (-3)2^7$

(c) $a_4 = 243, r = -3$

$a_n = (-9)(-3)^{n-1}$, $a_8 = (-9)(-3)^7$

(1) Study Guide # 1 : Pb.34, 37-50

34	A	44	E
37	B	45	B
38	A	46	B
38	B	47	A
40	B	48	C
41	C	49	C
42	A	50	E
43	B		

(2) Study Guide # 2 : Pb.34, 37-50

34	A	44	E
37	B	45	B
38	A	46	B
38	B	47	A
40	B	48	C
41	C	49	C
42	A	50	E
43	B		

(3) Study Guide # 3 : Pb.34, 37-50

34	A	44	E
37	B	45	B
38	A	46	B
38	B	47	A
40	B	48	C
41	C	49	C
42	A	50	E
43	B		