Practice TEST #3

(1) What is the least number of turning points an odd-degree polynomial function can have? An even-degree polynomial function?

(2) What is the maximum number of x intercepts the graph of a polynomial function of degree n can have?

(3) What is the maximum number of real solutions an n^{th}-degree polynomial equation can have?

(4) What is the least number of x intercepts the graph of a polynomial function of odd degree can have? of even degree?

(5) What is the least number of real solutions a polynomial equation equation of odd degree can have? an even degree?

(6) Either give an example of a polynomial with real coefficients that satisfies the given conditions or explain why such polynomial cannot exist.
   (a) $P(x)$ is a third-degree polynomial with one x intercept.
   (b) $P(x)$ is a fourth-degree polynomial with no x intercept.
   (c) $P(x)$ is a third-degree polynomial with no x intercept.
   (d) $P(x)$ is a fourth-degree polynomial with no turning points.

(7) If r is a real zero of a polynomial $P(x)$ with real coefficients, then r is also an x intercept for the graph of $P(x)$. What ia the difference between the graph of $P(X)$ at a real zero of odd multiplicity and at a real zero of even multiplicity.

(8) Find a polynomial of lowest degree, with leading coefficient 1, that has the indicated set of zeros. Leave the answer in the factor form. Indicate the degree of the polynomial.
   (a) 2(multiplicity 3), -1;
   (b) 0, 2,$-1+\sqrt{3}$,$-1-\sqrt{3}$;
   (c) 1(multiplicity 3),$-3+\sqrt{2}$, $-3-\sqrt{2}$;

(9) Find the domain and the x intercepts. Do not graph.
   (a) $f(x)=\dfrac{2x-4}{x+1}$.
   (b) $f(x)=\dfrac{x^2-1}{x^2-16}$.
   (c) $f(x)=\dfrac{x^2-x-6}{x^2-x-12}$.
   (d) $f(x)=\dfrac{x}{x^2+4}$.
   (e) $f(x)=\dfrac{x^2-4}{x-2}$.

(f) $f(x) = \dfrac{x-1}{x^2-1}$.

(10) Find all the asymptotes. Do not graph.

(a) $f(x) = \dfrac{2x}{x-4}$.

(b) $f(x) = \dfrac{2x^2+3x}{3x^2-48}$.

(c) $f(x) = \dfrac{2x}{x^4+1}$.

(d) $f(x) = \dfrac{6x^4}{3x^2-2x-5}$.

(e) $f(x) = \dfrac{x^2-1}{x+1}$.

(f) $f(x) = \dfrac{x+2}{x^2-4}$.

(11) Find an equation of a rational function whose graph has:
- vertical asymptote at $x = -1$
- an y-intercept $(0, 5)$
- a slant asymptote at $y = x - 1$

(12) Basic properties of the graph $f(x) = b^x$, $b > 0$, $b \neq 1$:

(a) All graphs pass through the point (0,1).
(b) All graphs are continuous, with no holes or jumps.
(c) The x axis is a horizontal asymptote.
(d) If $b > 1$, then b^x increases as x increases.
(e) If $0 < b < 1$, then b^x decreases as x increases.
(f) The function is one-to-one.

(13) If Kenya has a population of about 30,000,000 people and a doubling time of 19 years and if the growth continues at the same rate, find the population in: (A) 10 years; (B) 30 years. Compute answer to 2 significant digits.

(14) The use of insecticide DDT is no longer allowed in many countries because of its long-term adverse effects. If a farmer uses 25 pounds of active DDT, assuming its half-life is 12 years, how much still be active after: (A) 5 years? (B) 20 years? Compute answer to 2 significant digits.

(15) A couple just had a new child. How much should they invest now at 8.25% compounded daily in order to have $40,000 for the child's education 17 years from now? Compute answer to the nearest dollar.

(16) Find the function $f(x) = Cb^x$ if the graph of the function goes through the points $(0, \frac{1}{2})$ and $(\frac{1}{2}, 1)$.

(17) If the world population is about 6 billion people now and if the population grows continuously at an annual rate of 1.7%, what will the population be in 10 years? Compute the answer to 2 significant digits.

(18) For the one-to-one function given by $f(x) = \dfrac{x+2}{x-3}$.
- (a) Find $f^{-1}(x)$.
- (b) Find $f^{-1}(3)$.
- (c) Find $f^{-1}[f(x)]$.

(19) Given $f(x) = x^2 - 1$, $x \geq 0$:
- (a) Find the domain and range of f and f^{-1}.
- (b) Find $f^{-1}(x)$.
- (c) Find $f^{-1}(3)$.
- (d) Find $f^{-1}[f(4)]$.
- (e) Find $f^{-1}[f(x)]$.

(20) Let $f(x) = \sqrt{x+1}$, $g(x) = \dfrac{1}{x}$, and $h(x) = x + 3$. Find an equation defining each function and state the domain.

a)$f + g$ b) $f - g$ c)fg d) $\dfrac{f}{g}$ e) $f \circ g \circ h$

(21) Let $f(x) = (1,2),(2,3),(3,4),(4,5)$, $g(x) = (1,2),(2,3),(3,4),(4,5)$, and $h(x) = (1,0),(2,1),(3,2)$. Find each function and state the domain.

a)$f + g$ b) $f - g$ c)fg d) $\dfrac{f}{g}$ e) $f \circ g \circ h$

(22) Find two possible decompositions of $h(x) = \dfrac{1}{(x^3+x+1)^2}$

(23) Find one decomposition of $h(x) = (x-3)^3 + 2(x-3)^2 + 1$

(24) Find two possible decompositions of $h(x) = \sqrt{\dfrac{1}{x}}$

(25) Find one decomposition of $h(x) = \dfrac{2}{x^3} + \dfrac{3}{x^2} - \dfrac{1}{x}$

(26) Find two possible decompositions of $h(x) = \sqrt{x^2+1}$

(27) Find one decomposition of $h(x) = \dfrac{x+2}{(x+2)^2+1}$