Sections 3.5-3.7 MATH 122B

Consider the following table, and answer the below questions:

x	1	2	3	4
$f(x)$	2	4	1	3
$f'(x)$	-6	-7	-8	-9
$g(x)$	2	3	4	1
$g'(x)$	$2/7$	$3/7$	$4/7$	$5/7$

★ note, I've gone further w/ some trig simplification ... be aware you can (print off trig identity page + use)

1. If $h(x) = \cos(f(x))$ find $h'(3)$.

$$h'(x) = -\sin(f(x)) \cdot f'(x)$$
$$h'(3) = -\sin(f(3)) \cdot f'(3)$$
$$= -\sin(1) \cdot (-8)$$
$$= 8\sin(1)$$

2. If $h(x) = \arccos(g(x))$ find $h'(3)$.

$$h'(x) = \frac{-g'(x)}{\sqrt{1-(g(x))^2}}$$
$$h'(3) = \frac{-g'(3)}{\sqrt{1-(g(\ ))^2}} = \frac{-4/7}{\sqrt{1-4^2}}$$

D.N.E ... not a real value. $\frac{-4}{7\sqrt{-15}}$

3. If $h(x) = \sin(x) \cdot \cos(g(x))$ find $h'(2)$.

$$h'(x) = \cos x \cdot \cos(g(x)) + -\sin(g(x)) \cdot g'(x) \cdot \sin(x)$$
$$h'(2) = \cos 2 \cdot \cos(3) - \sin(3) \cdot \tfrac{3}{7} \cdot \sin(2)$$
$$= \tfrac{1}{2}\{\cos(-1) + \cos(5)\} - \tfrac{3}{7} \cdot \tfrac{1}{2}\{\cos(-1) - \cos(5)\}$$
$$= \tfrac{1}{2}\cos(1) + \tfrac{1}{2}\cos 5 - \tfrac{3}{14}\cos(1) + \tfrac{3}{14}\cos(5) = \boxed{\tfrac{2}{7}\cos(1) + \tfrac{5}{7}\cos(5)}$$

4. If $h(x) = \ln(f(x))$ find $h'(4)$.

$$h'(x) = \frac{f'(x)}{f(x)} \qquad h'(4) = \frac{f'(4)}{f(4)} = \frac{-9}{3} = -3$$

Find the derivative of the following: (Make sure you can simplify 'enough'... if you were to be asked to solve $f'(x) = 0$ you want a factored form of $f'(x)$ get it to this form as best as you can.)

1. $f(\theta) = \cos\theta \sin\theta$ $f'(\theta) = -\sin\theta\sin\theta + \cos\theta\cos\theta$

$$= \cos^2\theta - \sin^2\theta$$
$$= \cos(2\theta)$$ (★double angle)

2. $g(x) = \sqrt{(\sin(2x))^3} = \{\sin(2x)\}^{3/2}$

$$g'(x) = \tfrac{3}{2}\{\sin(2x)\}^{1/2} \cdot 2 = 3\sqrt{\sin(2x)}$$

3. $f(x) = e^{-2x} \cdot \cot x$ $f'(x) = -2e^{-2x} \cdot \cot x + {}^{-}\csc^2 x \cdot e^{-2x}$

$$= -e^{2x}\left[2\cot x + \csc^2 x\right]$$
$$= -e^{2x}\left[2\cot x + 1 + \cot^2 x\right]$$
$$= -e^{2x}(\cot x + 1)^2$$

4. $h(x) = \sin(\sec x + \tan x)$

$$h'(x) = \cos(\sec x + \tan x)\{\sec x \tan x + \sec^2 x\}$$
$$= \sec x \cdot \cos(\sec x + \tan x) \cdot (\tan x + \sec x)$$

5. $f(x) = \sqrt{\dfrac{1 - \sec x}{1 - \csc x}}$

$$f'(x) = \frac{1}{2}\left\{\frac{1-\sec x}{1-\csc x}\right\}^{-1/2}\left\{\frac{(1-\csc x)(-\sec x\tan x) - (1-\sec x)(+\csc x\cot x)}{(1-\csc x)^2}\right\}$$

$$= \frac{1}{2}\,\frac{1}{(1-\sec x)^{1/2}} \cdot \left[\left(1 - \frac{1}{\sin x}\right)\left(-\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}\right) - \left(1 - \frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}\right)\right]\frac{1}{(1-\csc x)^{5}}$$

$$= \frac{1}{2}\,\frac{1}{(1-\sec x)^{1/2}(1-\csc x)^{3/2}}\left[-\frac{\sin x}{\cos^2 x} + \frac{1}{\cos^2 x} - \frac{\cos x}{\sin^2 x} + \frac{1}{\sin^2 x}\right]$$

$$= \frac{1}{2}\left(\frac{1}{(1-\sec x)^{1/2}(1-\csc x)^{3/2}}\right)\left(-\sec x\tan x + \sec^2 x - \cot x\csc x + \csc^2 x\right)$$

6. $g(x) = \dfrac{\tan^2 x + 1}{\cot^2 x + 1}$

$$g'(x) = \frac{(\cot^2 x + 1)(2\tan x \sec^2 x) - (\tan^2 x + 1)(2\cot x(-\csc^2 x))}{(\cot^2 x + 1)^2}$$

$$= \frac{\csc^2 x\,(2\tan x \sec^2 x) + \sec^2 x\,(2\cot x \csc^2 x)}{(\csc^2 x)^2}$$

$$= \frac{2\sec^2 x\,(\tan x + \cot x)}{\csc^2 x}$$

★ What would be a different way to do this? ★

7. $h(x) = \cos(\arctan 3x)$

$$h'(x) = -\sin(\arctan 3x) \cdot \frac{3}{(3x)^2 + 1}$$

8. $f(x) = \ln(x^2 + 4x + 7 + \sin x)$

$$f'(x) = \frac{2x + 4 + \cos x}{x^2 + 4x + 7 + \sin x}$$

9. $f(x) = \ln(\ln x) + \ln(\ln 2)$

$$f'(x) = \frac{1}{\ln x} \cdot \frac{1}{x} + 0 = \frac{1}{x \ln x}$$

10. $f(x) = \cos(\ln x)$

$$f'(x) = -\sin(\ln x) \cdot \frac{1}{x}$$

11. $f(x) = \arctan\left(\frac{x}{1+x}\right)$ $f'(x) = \frac{1}{\left(\frac{x}{1+x}\right)^2 + 1}\left(\frac{(1+x)\cdot 1 - x(1)}{(1+x)^2}\right)$

$= \frac{1}{x^2 + (1+x)^2}$

$= \frac{1}{2x^2 + 2x + 1}$

12. $f(x) = x \arcsin x$

$f'(x) = 1 \cdot \arcsin x + \frac{1}{\sqrt{1-x^2}} \cdot x$

13. $f(x) = e^{\arctan x}$ $f'(x) = \frac{1}{x^2+1} \cdot e^{\arctan x}$

14. $f(x) = \cos(4)^{\arccos x}$ $f'(x) = \ln(\cos 4) \cdot \frac{-1}{\sqrt{1-x^2}} \cdot \cos(4)^{\arccos x}$

Use implicit differentiation to find $\frac{dy}{dx}$, assume a, b and c are constants.

1. $xy + x + y = a$ $1 \cdot y + \frac{dy}{dx} \cdot x + 1 + \frac{dy}{dx} = 0$

$\frac{dy}{dx}[x + 1] = -1 - y$

$\frac{dy}{dx} = \frac{-1-y}{x+1}$

2. $x^{2/3} + y^{2/3} = a^{2/3}$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\cdot\frac{dy}{dx} = 0$$

$$\frac{2}{3}y^{-1/3}\cdot\frac{dy}{dx} = \frac{-2}{3}x^{-1/3}$$

$$\frac{dy}{dx} = \frac{\frac{-2}{3}x^{-1/3}}{\frac{2}{3}y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}}$$

3. $\arctan(x^2y) = xy^2$

$$\frac{1}{(x^2y)^2+1}\left(2xy + x^2\cdot\frac{dy}{dx}\right) = 1\cdot y^2 + 2y\frac{dy}{dx}\cdot x$$

$$\frac{2xy}{x^4y^2+1} - y^2 = 2xy\cdot\frac{dy}{dx} - \frac{x^2}{x^4y^2+1}\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\frac{2xy}{x^4y^2+1} - y^2}{2xy - \frac{x^2}{x^4y^2+1}}$$

$$\frac{dy}{dx} = \boxed{\frac{2xy - y^2(x^4y^2+1)}{2xy(x^4y^2+1) - x^2}}$$

4. $e^{\cos y} = x^3 \arctan y$

$$e^{\cos y}\cdot(-\sin y)\frac{dy}{dx} = 3x^2\arctan y + x^3\cdot\frac{1}{y^2+1}\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx}\left[-\sin y\, e^{\cos y} - \frac{x^3}{y^2+1}\right] = 3x^2\arctan y$$

$$\frac{dy}{dx} = \boxed{\frac{-3x^2\arctan y}{\sin y\, e^{\cos y} - \frac{x^3}{y^2+1}}}$$

5. $x\ln y + y^3 = \ln x$

$$1\cdot\ln y + x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + 3y^2\cdot\frac{dy}{dx} = \frac{1}{x}$$

$$\ln y - 1/x = \frac{dy}{dx}\left(\frac{x}{y} + 3y^2\right)$$

$$\frac{dy}{dx} = \frac{\ln y - 1/x}{x/y - 3y^2}$$

$$= \boxed{\frac{yx\ln y - y}{x^2 - 3xy^3}}$$

6. $(x-a)^2 + y^2 = b$

$$2(x-a)\cdot 1 + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2(x-a)$$

$$\frac{dy}{dx} = \frac{-2(x-a)}{2y}$$

$$= \frac{a-x}{y}$$

7. $ax + bx^2 + cy + ay^2 = a + \arccos x$

$$a + 2b + c\frac{dy}{dx} + 2ay\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx}(c+2ay) = \frac{-1}{\sqrt{1-x^2}} - a - 2b$$

$$\frac{dy}{dx} = -\frac{1+(a+2b)\sqrt{1-x^2}}{(c+2ay)\sqrt{1-x^2}}$$

8. $e^{x^2} + \ln y = 0$

$$2xe^{x^2} + \frac{1}{y}\cdot\frac{dy}{dx} = 0$$

$$\frac{2xe^{x^2}}{-1/y} = \frac{dy}{dx}$$

$$-2yxe^{x^2} = dy/dx$$