*If anything looks wrong please let me know!

Product/Quotient/Chain Review MATH 122B

Consider the following table, and answer the below questions: (Be aware this can be asked in many different ways, with graphs, lists, part of the function given... similar to an exam question)

x	1	2	3	4
$f(x)$	2	4	1	3
$f'(x)$	-6	-7	-8	-9
$g(x)$	2	3	4	1
$g'(x)$	2/7	3/7	4/7	5/7

(a) If $h(x) = f(x) \cdot g(x)$ find $h'(1)$.

$h'(x) = f'(x)g(x) + g'(x)f(x)$

$h'(1) = f'(1)g(1) + g'(1)f(1)$

$= -6(2) + (3/7)(2)$

$= -12 + 4/7 = \boxed{-80/7}$

(b) If $h(x) = \dfrac{f(x)}{g(x)}$ find $h'(3)$.

$h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$h'(3) = \dfrac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2} = \dfrac{4(-8) - 1(4/7)}{(4)^2} = \boxed{-57/28}$

(c) If $h(x) = f(g(x))$ find $h'(2)$.

$h'(x) = f'(g(x))\,g'(x)$

$h'(2) = f'(g(2))\,g'(2)$

$= f'(3)(3/7) = (-8)(3/7) = \boxed{-24/7}$

(d) If $h(x) = f(x) + g(x)$ find $h'(4)$.

$f'(x) + g'(x) = h'(x)$

$h'(4) = f'(4) + g'(4) = -9 + 5/7 = \boxed{-58/7}$

Find the derivative of the following: (Make sure you can simplify 'enough'... if you were to be asked to solve $f'(x) = 0$ you want a factored form of $f'(x)$ get it to this form as best as you can.)

1. $f(x) = \frac{3}{x^4} - \frac{7}{x^3} + \sqrt{x} + \frac{10}{\sqrt[4]{x}}$ $= 3x^{-4} - 7x^{-3} + x^{1/2} + 10x^{-1/4}$

$f'(x) = -12x^{-5} + 21x^{-4} + \frac{1}{2}x^{-1/2} - \frac{10}{4}x^{-5/4}$

★ you should be able to rewrite w/ positive exponents + roots.

2. $f(x) = x^{-1/2} - 14x^{-3/2}$

$f'(x) = \frac{-1}{2}x^{-3/2} - 14(-3/2)x^{-3/2-2/2}$

$= \frac{-1}{2}x^{-3/2} + 21x^{-5/2}$

3. $f(x) = \frac{x^3+5}{x}$ ★ can do this 2 ways, power or quotient rule!

$= x^2 + 5x^{-1}$

$f'(x) = 2x - 5x^{-2}$

4. $f(x) = (8x^2 - 4x)^3$

$f'(x) = 3(8x^2-4x)^2(16x-4)$

5. $f(x) = 4x^3 - 6x^{-2}$

$f'(x) = 12x^2 + 12x^{-3}$

6. $f(x) = (x+1)(\sqrt{x} - 2)$ $= (x+1)(x^{1/2} - 2)$

$f'(x) = 1(\sqrt{x} - 2) + (\frac{1}{2}x^{-\frac{1}{2}})(x+1)$

$= x^{1/2} - 2 + \frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2}$

$= \frac{3}{2}x^{1/2} - 2 + \frac{1}{2}x^{-1/2}$

7. $f(x) = 5(3x^2 + 2)(2x - 1)(x^2 + 1)$

$f'(x) = 5(6x)(2x-1)(x^2+1) + 5(2)(3x^2+2)(x^2+1) + 5(2x)(3x^2+2)(2x-1)$

$= 10\{15x^4 + 3x^2 - 6x^3 + x + 2\}$

* probably need more work to get this answer... On hmk + test this is not enough work! (will receive no credit if this is given)

8. $f(x) = (x^{-1} - x^{-3})\left(3x^{-1} + \frac{x^4}{6}\right)$

$f'(x) = (-x^{-2} + 3x^{-4})(3x^{-1} + \frac{1}{6}x^4) + (-3x^{-2} + \frac{4}{6}x^3)(x^{-1} - x^{-3})$

9. $f(x) = \dfrac{6x + 1}{3x + 10}$

$f'(x) = \dfrac{(3x+10)6 - (6x+1)3}{(3x+10)^2} = \dfrac{18x + 60 - 18x - 3}{(3x+10)^2}$

$= \dfrac{57}{(3x+10)^2}$

10. $f(x) = 10 \cdot \dfrac{x^{2.2}}{x^{3.2} + 5}$

$f'(x) = 10 \cdot \dfrac{(x^{3.2}+5)2.2x^{1.2} - x^{2.2}(3.2x^{2.2})}{(x^{3.2}+5)^2}$

$= 10 \cdot \dfrac{2.2x^{4.4} + 11x^{1.2} - 3.2x^{4.4}}{(x^{3.2}+5)^2}$

$= \dfrac{-10x^{4.4} + 110x^{1.2}}{(x^{3.2}+5)^2}$

11. ~~$f(x) = \dfrac{(3x^2 + 1)(2x - 1)}{5x + 4}$~~

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11. $f(x) = \dfrac{(3x^2+1)(2x-1)}{5x+4}$ $f'(x) = \dfrac{(5x+4)\{6x(2x-1)+2(3x^2+1)\} - (3x^2+1)(2x-1)5}{(5x+4)^2}$

$= \dfrac{(5x+4)(12x^2-6x+6x^2+1) - 5(6x^3-3x^2+2x-1)}{(5x+4)^2}$

$= \dfrac{(5x+4)(18x^2-6x+1) - 30x^3+15x^2-10x+5}{(5x+4)^2}$

$= \dfrac{(90x^3-30x^2+5x+72x^2-24x+4) - 30x^3+15x^2-10x+5}{(5x+4)^2}$

$= \dfrac{60x^3+57x^2-29x+9}{(5x+4)^2}$

12. $f(x) = \dfrac{x^2-4}{x^3} = x^{-1} - 4x^{-3}$

$f'(x) = -x^{-2} + 12x^{-4}$

13. $f(x) = \dfrac{(2x^2+3)(5x+2)}{6x-7}$

$f'(x) = \dfrac{(6x-7)(4x(5x+2)+5(2x^2+3)) - (2x^2+3)(5x+2)6}{(6x-7)^2}$

$= \dfrac{120x^3-186x^2-56x-141}{(6x-7)^2}$

14. $f(x) = (8x^4-5x^2+1)^4$

$f'(x) = 4(8x^4-5x^2+1)^3(32x^3-10x)$

$= 8x(16x^2-5)(8x^4-5x^2+1)^3$

15. $f(x) = (3x^4+1)^4(x^3+4)$

$f'(x) = 4(3x^4+1)^3(12x^3)(x^3+4) + 3x^2(3x^4+1)^4$

$= 3x^2(3x^4+1)^3\{4\cdot 4x(x^3+4) + (3x^4+1)\}$

$= 3x^2(3x^4+1)^3\{16x^4+64x+3x^4+1\}$

$= 3x^2(3x^4+1)^3(19x^4+64x+1)$

16. $f(x) = \dfrac{(5x-6)^4}{3x^2+4}$

$f'(x) = \dfrac{(3x^2+4)4(5x-6)^3(5) - (5x-6)^4(6x)}{(3x^2+4)^2}$

$= \dfrac{(5x-6)^3\{(3x^2+4)20 - 6x(5x-6)\}}{(3x^2+4)^2}$

$= \dfrac{(5x-6)^3\{60x^2+80-30x^2+12x\}}{(3x^2+4)^2}$ ~~A~~just realized I can pull out a 2!

$= \dfrac{2(5x-6)^3\{15x^2+6x+40\}}{(3x^2+4)^2}$

17. $f(x) = -2(12x^2+5)^{-6}$

$f'(x) = 12(12x^2+5)^{-7}(24x)$

$= 288x(12x^2+5)^{-7}$

18. $f(x) = 12x(2x^4+5)^{3/2}$

$f'(x) = 12x\left(\frac{3}{2}\right)(2x^4+5)^{1/2}(8x^3)$

$= 144x^4(2x^4+5)^{1/2}$

19. $f(x) = \dfrac{x^2+4x}{(3x^3+2)^4}$

$$f'(x) = \frac{(3x^3+2)^4(2x+4) - (x^2+4x)4(3x^3+2)^3(9x^2)}{(3x^3+2)^8}$$

$$= \frac{2(3x^3+2)^3\{(3x^3+2)(x+2) - 9x^2(2)(x^2+4x)\}}{(3x^3+2)^8}$$

$$= \frac{2\{3x^4+6x^3+2x+4 - 18x^4 - 72x^3\}}{(3x^3+2)^5}$$

$$= \frac{2\{-15x^4 - 66x^3 + 2x + 4\}}{(3x^3+2)^5}$$

20. $f(x) = -3\sqrt{7x^3-1}$ $= -3(7x^3-1)^{1/2}$

$$f'(x) = \frac{-3}{2}(7x^3-1)^{-1/2}(21x^2)$$

$$= \frac{-63}{2}x^2(7x^3-1)^{-1/2}$$

21. $f(x) = \sqrt{x^3-6x^2+9x+1}$ $= (x^3-6x^2+9x+1)^{1/2}$

$$f'(x) = \frac{1}{2}(x^3-6x^2+9x+1)^{-1/2}(3x^2-12x+9)$$

$$= \frac{3}{2}(x-1)(x-3)(x^3-6x^2+9x+1)^{-1/2}$$

22. $f(x) = e^{\sqrt{x^3-6x^2+9x+1}}$ * note the derivative of this power is the question above!

$$f'(x) = \frac{3}{2}(x-1)(x-3)(x^3-6x^2+9x+1)\,e^{(x^3-6x^2+9x+1)^{1/2}}$$

23. $f(x) = e^{5x^2+3^x+e^{2x+5}}$

$$f'(x) = (10x + \ln 3\,(3^x) + 2e^{2x+5})\,e^{5x^2+3^x+e^{2x+5}}$$

24. $f(x) = (x^6 + 4x)^{100}(x^7 + 7^x)\sqrt[7]{2 - x + x^7}$ $= (x^6+4x)^{100}(x^7+7^x)(2-x+x^7)^{1/7}$

$$f'(x) = 100(x^6+4x)^{99}(x^7+7^x)(2-x+x^7)^{1/7} + (7x^6+(\ln 7)7^x)(x^6+4x)^{100}(2-x+x^7)^{1/7}$$
$$+\tfrac{1}{7}(2-x+x^7)^{-6/7}(x^6+4x)^{100}(x^7+7^x)$$

$$= (x^6+4x)^{99}(2-x+x^7)^{-6/7}\left\{100(x^7+7^x)(2-x+x^7) + (7x^6+(\ln 7)7^x)(x^6+4x)(2-x+x^7)\right.$$
$$\left. + \tfrac{1}{7}(x^6+4x)(x^7+7^x)\right\}$$

25. $f(x) = \dfrac{(x^6 + 4x)^{100}}{(x^7 + 7^x)\sqrt[7]{2 - x + x^7}}$

$$f'(x) = \frac{(x^7+7^x)(2-x+x^7)^{1/7}\left(100(x^6+4x)^{99}(6x^5+4)\right) - (x^6+4x)^{100}\left\{(7x^6+(\ln 7)7^x)(2-x+x^7)^{1/7}\right.}{(x^7+7^x)^2(2-x+x^7)^{2/7}}$$
$$\left. + \tfrac{1}{7}(2-x+x^7)^{-6/7}(-1+7x^6)(x^7+7^x)\right\}$$

$$= \frac{(x^6+4x)^{99}(2-x+x^7)^{-6/7}\left[(x^7+7^x)(2-x+x^7)100(6x^5+4) - (x^6+4x)\left\{(7x^6+(\ln 7)7^x)(2-x+x^7)\right.\right.}{(x^7+7^x)^2(2-x+x^7)^{2/7}}$$
$$\left.\left. + \tfrac{1}{7}(-1+7x^6)(x^7+7^x)\right\}\right]$$

$$= \frac{(x^6+4x)^{99}\left[(x^7+7^x)(2-x+x^7)100(6x^5+4) - (x^6+4x)\left\{(7x^6+(\ln 7)7^x)(2-x+x^7)\right.\right.}{(x^7+7^x)^2(2-x+x^7)^{3/7}}$$
$$\left.\left. + 1/7(-1+7x^6)(x^7+7^x)\right\}\right]$$

26. $f(x) = e^{x^2}$

$$f'(x) = 2x\,e^{x^2}$$

27. $f(x) = 5^{x^2}$

$$f'(x) = (\ln 5)\,2x\,(5^{x^2})$$

28. $f(x) = x^2 \cdot 5^{x^3}$

$$f'(x) = 2x \cdot 5^{x^3} + \ln(5)3x^2 \cdot 5^{x^3} \cdot x^2$$
$$= 5^{x^3} \cdot x\left\{2 + 3x^3(\ln 5)\right\}$$

30. $f(x) = 15^{\sqrt{x^3+x}}$ $f'(x) = \frac{1}{2}(x^3+x)^{-1/2}(3x^2+1)\ln(15)\,15^{(x^3+x)^{1/2}}$

31. $f(x) = \left(\frac{8x-x^6}{x^4+4}\right)^{-4/5}$ $f'(x) = -\frac{4}{5}\left(\frac{8x-x^6}{x^4+4}\right)^{-9/5}\left(\frac{(x^4+4)(8-6x^5)-(8x-x^6)(4x^3)}{(x^4+4)^2}\right)$

$$= -\frac{4}{5}\left(\frac{x^4+4}{8x-x^6}\right)^{9/5}\left(\frac{8x^4-6x^9+32-24x^5-32x^4+4x^9}{(x^4+4)^2}\right)$$

$$= -\frac{4}{5}\,\frac{(x^4+4)^{9/5}}{(8x-x^6)^{9/5}}\cdot\frac{(-2x^9-24x^5-24x^4+32)}{(x^4+4)^2}$$

$$= \frac{8\,(x^9+12x^5+12x^4-16)}{5\,(8x-x^6)^{9/5}\,(x^4+4)^{1/5}}$$

32. $f(x) = 10\left(1+\left(2-(6+7x^4)^9\right)^3\right)^5$

$f'(x) = 50\left(1+(2-(6+7x^4)^9)^3\right)^4\left(0+3(2-(6+7x^4)^9)^2\right)\left(0-9(6+7x^4)^8\right)(28x^3)$

★this one was hard for me!