3 3 3 1

$7\tan\theta + \pi\theta + \frac{2}{5}\theta^{5/2} + C$

Exam IV
Summer 2014

100/100

MATH 122B
Sections 5.1 – 7.1

Name: Key.

You must *show work* and *simplify all answers* to receive full credit. If your work does not support your answer you will receive **no credit!**

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1. Let $v(t) = \dfrac{1}{\ln t}$ be the velocity of an object in meters per second moving along a straight line. Use a right-hand Riemann sum with $n = 3$ to approximate to the nearest hundredth the total distance traveled by the object over the interval $2 \le t \le 6$ (include units!).

$n=3$

$\Delta t = \frac{6-2}{3} = \frac{4}{3}$ 2

$t_0 = 2$
$t_1 = 10/3$
$t_2 = 14/3$
$t_3 = 6$
2

$$\underset{2\le t\le 6}{\text{Distance traveled}} \approx v(t_1)\Delta t + v(t_2)\Delta t + v(t_3)\Delta t$$

$$= \left(\frac{1}{\ln(10/3)}\right)\left(\frac{4}{3}\right) + \left(\frac{1}{\ln(14/3)}\right)\left(\frac{4}{3}\right) + \left(\frac{1}{\ln 6}\right)\left(\frac{4}{3}\right)$$ 6

$$\approx (0.831)(4/3) + (0.649)(4/3) + (0.558)(4/3)$$

$$\approx 1.107 + 0.866 + 0.744$$ 2

$$= \boxed{2.72} \text{ meters}$$

-1 units

-1 for rounding.

-9 for $\frac{1}{\ln 3} + \frac{1}{\ln 4} + \frac{1}{\ln 5}$ or similar

-7 if the above, but got Δt.

-3 for left-sum

-6 for 2 rec.

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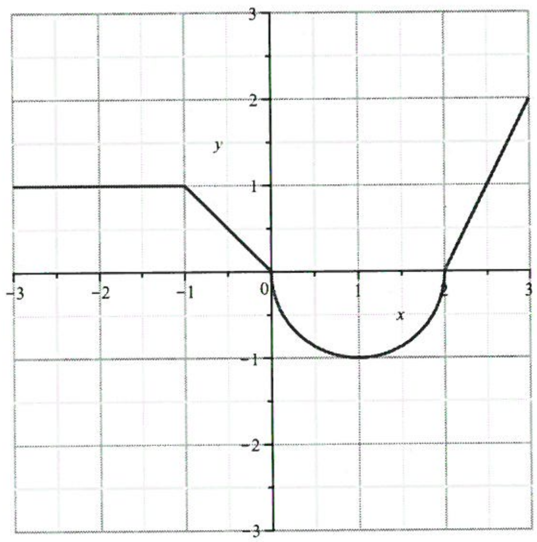
2. Let $f(x)$ be given by the graph seen below,

Graph of $f(x)$:



Determine the exact value of $\int_{-3}^{3} 2f(x)+1\, dx$.

$$\int_{-3}^{3} 2f(x)+1\,dx = 2\int_{-3}^{3} f(x)dx + \int_{-3}^{3} 1\,dx$$

$$= \underbrace{2\left(\overset{2}{2} + \overset{2}{\tfrac{1}{2}(1)} + \overset{2}{\left(-\tfrac{\pi}{2}\right)} + \overset{2}{\tfrac{1}{2}(2)}\right)}_{6} + \underbrace{x\Big|_{-3}^{3}}_{2}$$

$$= 2\left(2+\tfrac{1}{2}-\tfrac{\pi}{2}+1\right) + (3-(-3))$$

$$= 2\left(\tfrac{7}{2}-\tfrac{\pi}{2}\right) + 6$$

$$= 7 - \cancel{2}\pi + 6$$

$$= 13 - \cancel{2}\pi$$

$$= \boxed{13-\pi}$$

2

-6 for attempt to shift

~~1~~ miss (x2)

-2 for Approx

-3 for $\int_{-3}^{3} 2f(x)dx + 1$

2

-6 for just $\int f(x)$

12

3. Suppose you just bought a brand new speedboat. Your speed, given in miles per hour, on your initial voyage is given by

$$r(t) = 12t - \frac{4}{1+t^2} - \frac{1}{t}$$

where t is measured in hours. If you know that the speedboat can travel 6 miles per gallon of gasoline, about how many gallons did you use between hour 1 and hour 3 of your voyage?

Distance traveled between $1 \le t \le 3$ $= \int_1^3 12t - \frac{4}{1+t^2} - \frac{1}{t}\, dt$

$= 6t^2 - 4\arctan t - \ln|t| \Big|_1^3$ 6

$= [6(3)^2 - 4\arctan(3) - \ln(3)] - [6(1)^2 - 4\arctan(1) - \ln(1)]$

$\approx [54 - 4(1.249) - 1.099] - [6 - 4(0.785) - 0]$ 3

$= 47.905 - 2.858$

$= 45.047$

Gallons used $\approx \frac{45.047}{6} \approx \boxed{7.5 \text{ gallons}}$ 3

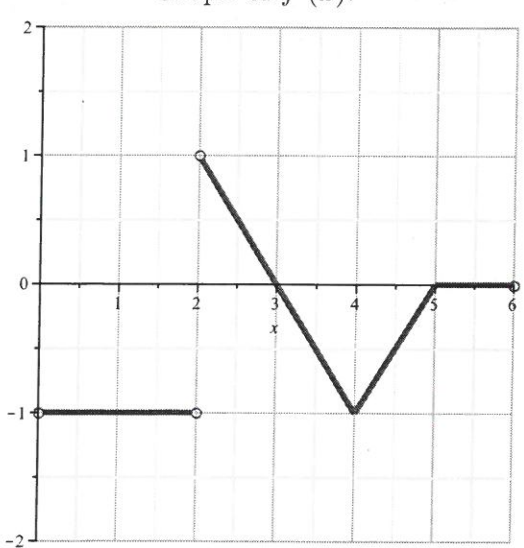
-2 for mult

-3 not w/6.

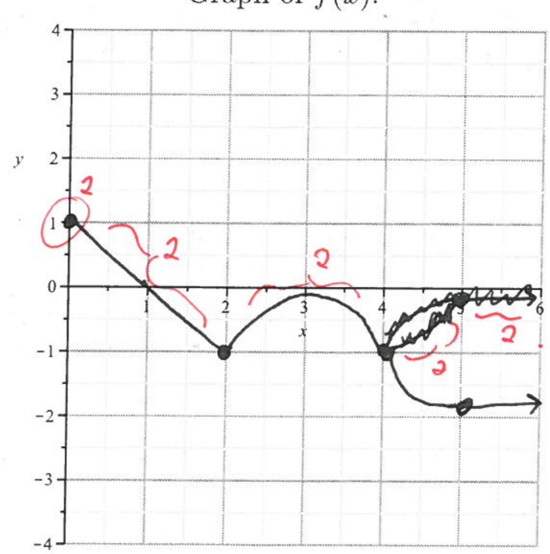
-1 for approx

10. 4. Using the graph of $f'(x)$ seen below and the fact that $f(0) = 1$ sketch the graph of $f(x)$.

Graph of $f'(x)$:



Graph of $f(x)$:



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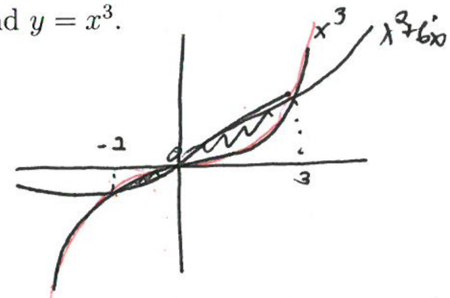
5. Find the area of the region between the curves $y = x^2 + 6x$ and $y = x^3$.



$$x^3 = x^2 + 6x$$
$$x^3 - x^2 + 6x = 0$$
$$x(x^2 - x + 6) = 0$$
$$x(x-3)(x+2) = 0$$
$x = 0, -2, 3$ pts. of intersection.

3

$$\int_{-2}^{0} x^3 - (x^2+6x)\,dx + \int_0^3 x^2+6x - x^3\,dx$$ 3

-2 for not switching functions.

-5 for only one interval.

$$= \int_{-2}^{0} x^3 - x^2 - 6x\,dx + \int_0^3 -x^3 + x^2 + 6x\,dx$$

$$= \left(\frac{x^4}{4} - \frac{x^3}{3} - 3x^2\Big|_{-2}^{0}\right) + \left(-\frac{x^4}{4} + \frac{x^3}{3} + 3x^2\Big|_0^3\right)$$ 3

$$\boxed{\frac{253}{12} \approx 21.0833}$$

3

$$= \left[\left(\frac{0}{4} - \frac{0}{3} - 3(0)\right) - \left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - 3(-2)^2\right)\right] + \left[\left(-\frac{(3)^4}{4} + \frac{(3)^3}{3} + 3(3)^2\right) - \left(-\frac{0}{4} + \frac{0}{3} + 3(0)\right)\right]$$

$$= -\left(\frac{16}{4} + \frac{8}{3} - 12\right) + \left(-\frac{81}{4} + \frac{27}{3} + 27\right)$$

16/3 = 64/12, 63/4 = 189/12

$$= -\left(\frac{48}{12} + \frac{32}{12} - \frac{144}{12}\right) + \left(-\frac{243}{12} + \frac{108}{12} + \frac{324}{12}\right) = -\left(\frac{-64}{12}\right) + \left(\frac{189}{12}\right) = \frac{64}{12} + \frac{189}{12} = \boxed{\frac{253}{12}}$$

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6. Determine $\int \frac{7}{\cos^2\theta} + \pi + \theta\sqrt{\theta}\,d\theta$.

$$\int \frac{7}{\cos^2\theta} + \pi + \theta\sqrt{\theta}\,d\theta = \int 7\sec^2\theta + \pi + \theta^{3/2}\,d\theta$$

$$= \boxed{7\tan\theta + \pi\theta + \frac{2}{5}\theta^{5/2} + C}$$

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7. Find the solution of the initial value problem,

$$\frac{dz}{dw} = 2 + \sin w, \qquad z(0) = 5.$$

$z = 2w - \cos w + C$ 4

$5 = 2(0) - \cos(0) + C$
$5 = -1 + C$
$6 = C$ 4

$\boxed{z = 2w - \cos w + 6}$ 2

-1 for +C

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8. Compute the following derivative:

$$\frac{d}{dx}\int_2^{x^3} \sin(t^2)dt.$$

$F(x) = \int_2^x \sin(t^2)\,dt$ AND $F'(x) = \sin(x^2)$ by 2nd fund. thm. 4

$\frac{d}{dx}\left(\int_2^{x^3} \sin(t^2)dt\right) = \underbrace{\frac{d}{dx}\left(F(x^3)\right) = F'(x^3)3x^2}_{4} = \sin((x^3)^2)3x^2$

$= \boxed{3x^2\sin(x^6)}$ 2

-1 for "+C"

-3 for cos

-4 for no $3x^2$.

-2 for $6x^5$

-8 for A $F(w)-F(0) = \int_0^w$ —

12

9. Evaluate the following integral:

$$\int_0^{3/2} \frac{6x-3}{\sqrt{2x+1}}\, dx.$$

$$\left.\begin{aligned} &\text{Let } u = 2x+1 \rightsquigarrow x = \frac{u-1}{2} \\ &du = 2dx \rightsquigarrow dx = \tfrac{1}{2}du \end{aligned}\right] 4$$

$$\int_0^{3/2} \frac{6x-3}{\sqrt{2x+1}}\,dx = \int_1^4 \frac{6\left(\frac{u-1}{2}\right)-3}{\sqrt{u}}\left(\tfrac{1}{2}du\right) = \frac{1}{2}\int_1^4 \frac{3(u-1)-3}{\sqrt{u}}\,du$$

$$= \frac{1}{2}\int_1^4 \frac{3u-6}{u^{1/2}}\,du \quad 4$$

-2 for not changing bounds (or dealing w/ them appropriately)

$$= \frac{1}{2}\int_1^4 3u^{1/2} - 6u^{-1/2}\,du$$

$$= \frac{1}{2}\left(3\left(\tfrac{2}{3}u^{3/2}\right) - 6\left(2u^{1/2}\right)\Big|_1^4\right)$$

$$= \frac{1}{2}\left(2u^{3/2} - 12u^{1/2}\Big|_1^4\right)$$

$$= \frac{1}{2}\left[\left(2(4)^{3/2} - 12(4)^{1/2}\right) - \left(2(1)^{3/2} - 12(1)^{1/2}\right)\right]$$

$$= \frac{1}{2}\left[\left(2(8) - 12(2)\right) - (2-12)\right]$$

-1 for not eval. at x=0

$$= \frac{1}{2}\left[16 - 24 - 2 + 12\right]$$

-5 for letting $du = \frac{u-1}{2}$.

$$= \frac{1}{2}[2]$$

$$= \boxed{1.} \quad 1$$

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