

Total 100pts.

Exam I  
Summer 2016

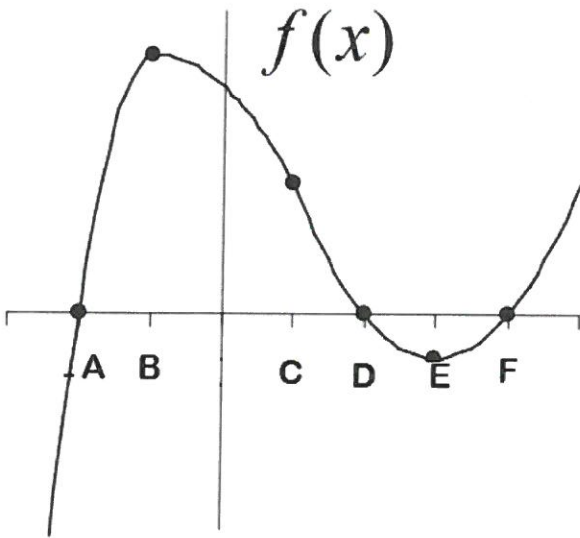
MATH 122B  
Sections 2.1 - 3.2

Key

Name: \_\_\_\_\_

You must *show work* and *simplify all answers* to receive full credit. Follow any specific directions in each question, if you use a method other than what is stated in that particular question, you will receive **no credit!** If your work does not support your answer, you will receive **no credit!**

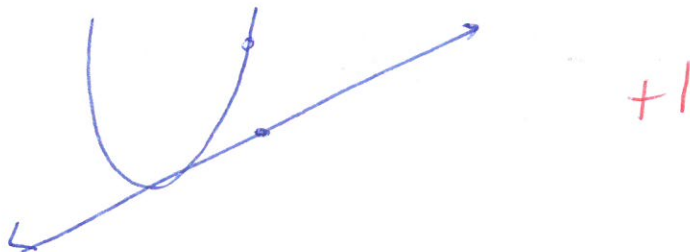
1. Using the following graph state the  $x$ -value(s) ( $A-E$ ) that satisfy each of the following conditions. (Write none, if no  $x$ -value does)



- (a)  $f''(x) = 0$  C +1
- (b)  $f''(x) < 0$  and  $f'(x) < 0$  none +1
- (c)  $f(x) \geq 0$ ,  $f'(x) > 0$  and  $f''(x) > 0$  F +1

2. (True/False) Let a  $f''(a) > 0$  for some real value  $a$ . If you were to approximate  $f(a+h)$  for some small value of  $h$  by evaluating the tangent line of  $f(x)$  at  $a$  this approximation will be greater than the actual value of  $f(a+h)$ .

- (a) TRUE
- (b) FALSE



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3. (Multiple Choice) Which of the following best describes a function with positive first derivative and negative second derivative?

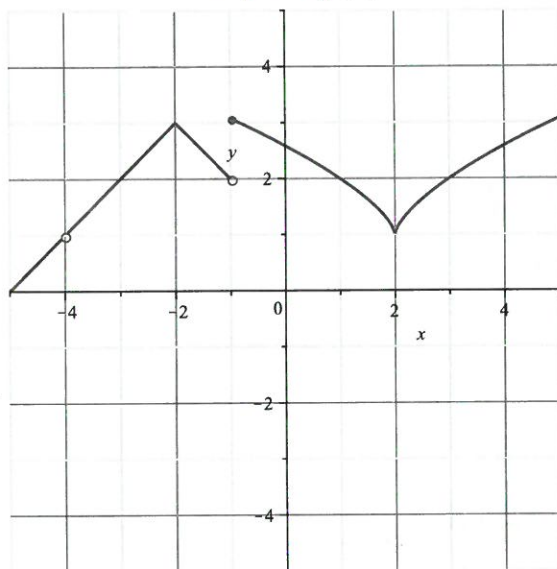
$f' > 0$   
 $f'' < 0$

- (a) A function that is decreasing at a constant rate.
- (b) A function that is decreasing at a decreasing rate.
- (c) A function that is decreasing at an increasing rate.
- (d) A function that is increasing at a constant rate.
- (e) A function that is increasing at a decreasing rate.
- (f) A function that is increasing at an increasing rate.
- (g) None of the above.

+1

4. Let  $f(x)$  be given by the graph shown below.

Graph of  $f(x)$



Use  $f(x)$  above to answer the following:

(a) For which  $x$ -values is  $f(x)$  not continuous?

$-4, -1$

+2

(b) For which  $x$ -values is  $f(x)$  not differentiable?

$-4, -2, 1, 2$

+4

(c) Is  $f'(x)$  increasing, decreasing or constant on the interval  $(2, 5)$ ?

decreasing

( $f$  is increasing) +2

(d) Is the sign of  $f''(-3)$  positive, negative or zero?

zero.

+2

5. If  $f(x) = x^x$  estimate the value of  $f'(3)$  using  $h = 0.001$ . Round your answer to three decimal places.

$$\frac{f(3+h) - f(3)}{h} = \frac{3.001^{3.001} - 3^3}{0.001} = 56.727$$

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6. The temperature,  $H$ , in degrees Celsius, of a cup of hot coffee placed on the kitchen counter is given by  $H = f(t)$ , where  $t$  is in minutes since the coffee was put on the counter.

(a) Is  $f'(t)$  positive or negative? Why?

+2 negative, coffee is cooling  
temp is decreasing  
+2

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(b) What are the units of  $\frac{dH}{dt}$ ?

+2 degrees Celsius / minutes  
+2

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(c) What is the **practical meaning** of  $f'(20) = -3$ ?

During the 20<sup>th</sup> minute the temp decreases 3°C.  
+1      +1      +1      13  
(or at 20 min temp dec 3°C/min)

7. Considering

$$\lim_{h \rightarrow 0} \frac{e^{(3+h)^2} - (3+h)^3 - e^{3^2} + 3^3}{h}$$

complete the following sentence.

The above expression would be used to find  $f'$ ( 3 ) when  $f(x) = \underline{e^{x^2} - x^3}$ .

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8. Find  $\frac{d^2z}{dy^2}$  for the equation defined by  $z = \frac{1}{2}y^3 + \frac{2}{y^2} - \frac{3}{4}y + 1$ .

$$\frac{dz}{dy} = \frac{3}{2}y^2 - 4y^{-3} - \frac{3}{4}$$

$\begin{matrix} +2 & +2 & +2 \end{matrix}$

$$= \frac{1}{2}y^3 + 2y^{-2} - \frac{3}{4}y + 1$$

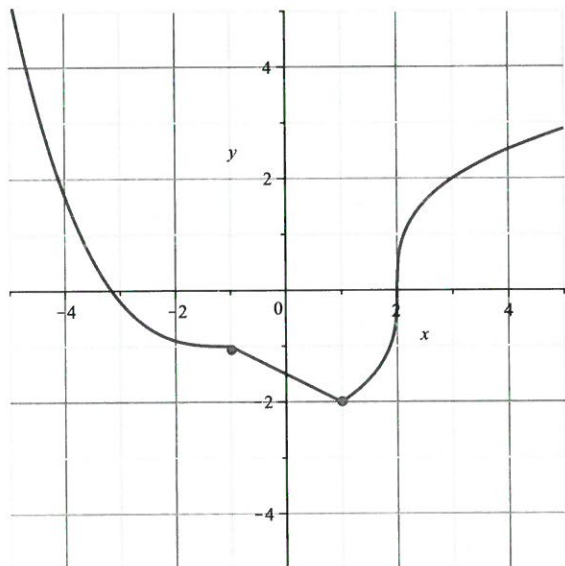
$$\frac{d^2z}{dy^2} = 3y + 12y^{-4}$$

$\begin{matrix} +2 & +2 \end{matrix}$

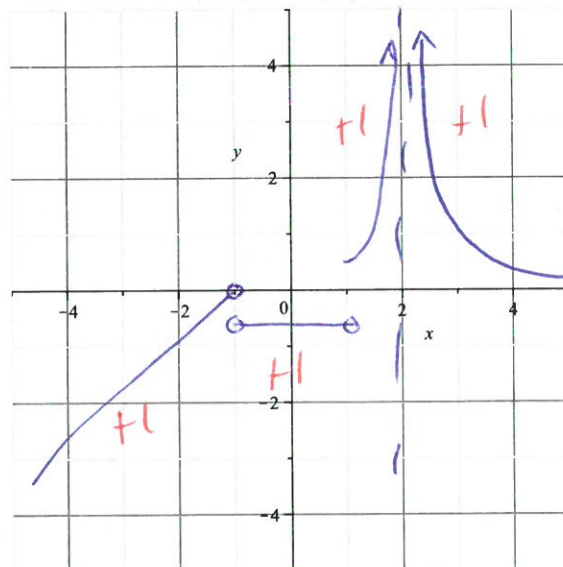
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9. Given the following graph of  $f(x)$ , sketch an approximate graph of  $f'(x)$ . (Note, you can always find  $f'$  exactly for the portion of the graph that is linear. For the other parts of  $f(x)$  you may roughly estimate.)

Graph of  $f(x)$ :



Graph of  $f'(x)$ :



+1 holes/asympt.

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10. Compute the derivative of  $k(x) = \pi^2 + \pi^x + x^\pi + \frac{\pi}{x^2}$ .

$$k' = 0 + (\ln \pi) \pi^x + \pi x^{\pi-1} + (-2) \pi x^{-3}$$

+1
+1
+1
+1

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11. Suppose  $h(x)$  is a differentiable function with  $h'(-2) = 5$ . If  $f(x) = 3h(x) + 1$ , determine  $f'(-2)$ .

$$f'(x) = 3h'(x) + 0 \quad +2$$

$$f'(-2) = 3h'(-2)$$

$$= 3(5) = 15 \quad +2$$

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12. Suppose  $f(x)$  is a differentiable function for all values of  $x$  with  $f(x)$  satisfying:

$$\lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h} = -3 \quad \text{and} \quad \lim_{x \rightarrow 6} f(x) = 7.$$

Write the equation of the tangent line to  $f(x)$  at  $x = 6$ .

$$y - y_1 = m(x - x_1)$$

$$y - 7 = -3(x - 6)$$

$$y = -3(x - 6) + 7$$

$$y = -3x + 18 + 7 \quad +2$$

$$y = -3x + 25$$

$y_1 = 7 \quad +2$   
 $x_1 = 6 \quad +2$   
 $m = -3 \quad +2$

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13. Use the **definition of the derivative** to compute the derivative of  $f(x) = 3x^2 + 2$ . (i.e., use limits!)

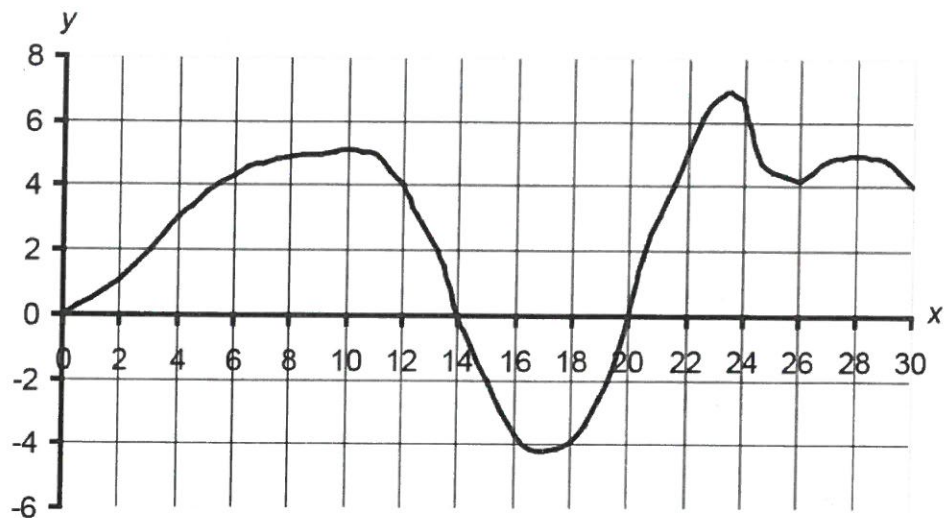
$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 + 2 - (3x^2 + 2)}{h} \quad +2 \\ &= \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) + 2 - 3x^2 - 2}{h} \quad +2 \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 2 - 3x^2 - 2}{h} \quad +2 \\ &= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} \quad +2 \\ &= \lim_{h \rightarrow 0} \frac{6x + 3h}{1} = 6x \quad +2\end{aligned}$$

14. Let  $g(x) = 7x^2 - x^{1/3}$ . Algebraically determine for which  $x$ -value(s) is  $g(x)$  not differentiable? (Show your work!)

$$g'(x) = 14x - \frac{1}{3}x^{-2/3} \quad +2$$

$g'(x)$  undefined at  $x=0$ .  $+2$

15. Arrange the following numbers from smallest (1) to largest (5) using the graph of  $f(x)$  below.



5  $\lim_{h \rightarrow 0} \frac{f(20+h) - f(20)}{h}$

3 The slope of  $f$  at  $x = 8$

1  $f(16)$

4 The average rate of change of  $f$  from  $x = 14$  to  $x = 24$

2  $\left. \frac{dy}{dx} \right|_{x=28}$

1 pt each

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16. Let  $s(t)$  denote the position of a particle at time  $t$  for  $0 \leq t \leq 10$ .

(a) What does the average velocity of  $s(t)$  between  $t = 2$  and  $t = 5$  represent **graphically**?

Slope of secant line

1/4

(b) What does the instantaneous rate of change of  $s(t)$  at  $t = 7$  represent **graphically**?

Slope of tan. line

1/4

(c) What is the **practical meaning** of  $s''(4)$ ?

acceleration at time  $t=4$

1/4