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**Exam II**
Summer 2014

MATH 122B
Sections 3.3 - 4.1

Name: Key

You must *show work* and *simplify all answers* to receive full credit. Follow any specific directions in each question, if you use a method other than what is stated in that particular question, you will receive **no credit!** If your work does not support your answer, you will receive **no credit!**

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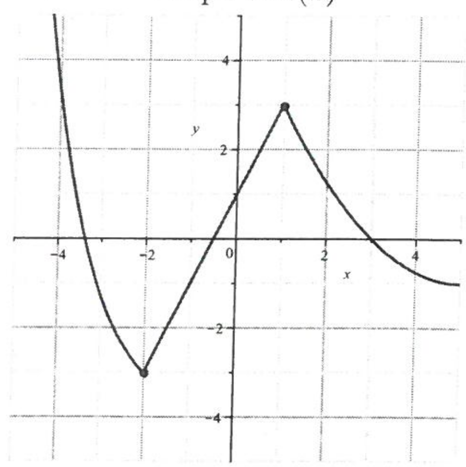
1. Assume the functions $f, g$, and $h$ are defined as follows:

$$f(x) = \frac{1}{2}x^2 - 4x,$$

$f'(x) = x - 4$

| $x$ | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $g(x)$ | 2 | 3 | 4 | 5 | 6 |
| $g'(x)$ | 7 | 10 | 5 | 2 | 3 |

Graph of $h(x)$




Use the functions $f, g$ and $h$ from above to determine the following:

(a) What is the slope of the tangent line to the graph of the function $(fg)(x)$ at the point $x = 2$?

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$(fg)'(2) = f'(2)g(2) + f(2)g'(2)$

$= (-2)(5) + (-6)(2) = -10 - 12 = \boxed{-22}$

-2 for not eval. @ 2

-3 for no prod/quo/chain.

(b) $\left(\frac{g}{h}\right)'(-1)$

4

$= \frac{h(-1)g'(-1) - g(-1)h'(-1)}{h(-1)^2} = \frac{(-1)(7) - (2)(2)}{(-1)^2} = \frac{-7-4}{1} = \boxed{-11}$

(c) $\left.\frac{d}{dx}4^{h(x)}\right|_{x=0}$

4

$= (\ln 4)4^{h(0)}h'(0) = (\ln 4)4'(2) = \boxed{8\ln 4}$

miss -2

(d) For which $x$-values is $\frac{d}{dx}4^{h(x)}$ undefined?

2

$\frac{d}{dx}4^{h(x)} = (\ln 4)4^{h(x)}h'(x)$ And $h'(x)$ is not defined at $\boxed{\begin{array}{l} x = -2 \; +1 \\ x = 1 \; +1 \end{array}}$



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2. Determine the $x$-value(s) for which the function $f(x) = x - \ln(10 + 3x)$ has a horizontal tangent line.

$f'(x) = 1 - \frac{1}{10+3x}(3)$ each deriv. is 4pts

$= 1 - \frac{3}{10+3x}$ 8

$0 = 1 - \frac{3}{10+3x}$ 2

$\frac{3}{10+3x} = 1$

$3 = 10+3x$

$-7 = 3x$

$\boxed{-7/3 = x}$ 2

, 3 for eval. @ 0.

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3. Differentiate $g(\theta) = \sin(\pi\theta) + \cosh(2\theta + 1) + \arctan(\theta^2)$.

$\boxed{g'(\theta) = \pi\cos(\pi\theta) + 2\sinh(2\theta+1) + \frac{2\theta}{1+\theta^4}}$

4 4 4

, 2 b. $\theta^2$.

Pink text.

$-\pi\sin(\pi\theta) + 2\cosh(2\theta+1) + \frac{2\theta}{1+\theta^4}$



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4. Let $f$ be a continuous function such that $f'(x) = 3x^4 - ax^3 + 2$ where $a$ is a fixed positive constant. Determine the inflection points of $f$. (Be sure to check your points are inflection points!)

$$f''(x) = 12x^3 - 3ax^2$$

$$= 3x^2(4x - a)$$ 3

-3 for finding $f'''$.

-6 for not justify.

$$0 = 3x^2(4x - a)$$

3

$$\left.\begin{array}{l} x = 0 \\ x = a/4 \end{array}\right\} \rightarrow \text{Possible inf. pts.}$$

− − +

6

| $f''<0$ | $f''=0$ | $f''<0$ | $f''=0$ | $f''>0$ |
|---|---|---|---|---|
| $-1$ | $0$ | $a/8$ | $a/4$ | $a$ |
| +2 | | +2 | | +2 |

2

$\boxed{x = a/4 \text{ is the inflection point.}}$

− 2 saying $x=0$

−2 if wrong concavity.

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5. Consider the curve defined by the equation $x^2 + 2xy + y^3 = 4$.

(a) Find $\frac{dy}{dx}$.
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$$\frac{d}{dx}(x^2+2xy+y^3) = \frac{d}{dx}(4)$$

$$2x + 2\left(y + x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$$ 6

$$2x + 2y + 2x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2x+3y^2) = -2x-2y$$ 2

$$\boxed{\frac{dy}{dx} = \frac{-2x-2y}{2x+3y^2}}$$ 2

(b) Find the local linearization of the curve at the point (1,1).
4

slope $= \left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-2(1)-2(1)}{2(1)+3(1)^2} = \frac{-2-2}{2+3} = \frac{-4}{5}.$ 2

point: $(1,1)$

$\boxed{y = 1 - \frac{4}{5}(x-1)}$ 2 or: $\boxed{y = -\frac{4}{5}x + \frac{9}{5}}$

-1 for wrong ad.

(c) Find an estimate for the $y$-value of the curve near the point $(1, 1)$ when $x = 1.5$.
2

$$y \approx -\frac{4}{5}\left(\frac{3}{2}\right) + \frac{9}{5}$$

$$= -\frac{6}{5} + \frac{9}{5}$$

$$= \boxed{\frac{3}{5} = .6}$$



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~~12~~ 8

6. Sketch a possible graph of a **continuous function**, $f(x)$, using the following information about the derivatives $f'(x)$ and $f''(x)$:

- $f'(-2)$ is undefined
- $f'(3) = 0$
- $f'(x) > 0$ for $-\infty < x < -2$
- $f'(x) < 0$ for $-2 < x < 3$
- $f'(x) > 0$ for $3 < x < \infty$

$f'>0$ | $f'$ und. | $f'<0$ | $f'=0$ | $f'>0$ ; $-2$, $3$

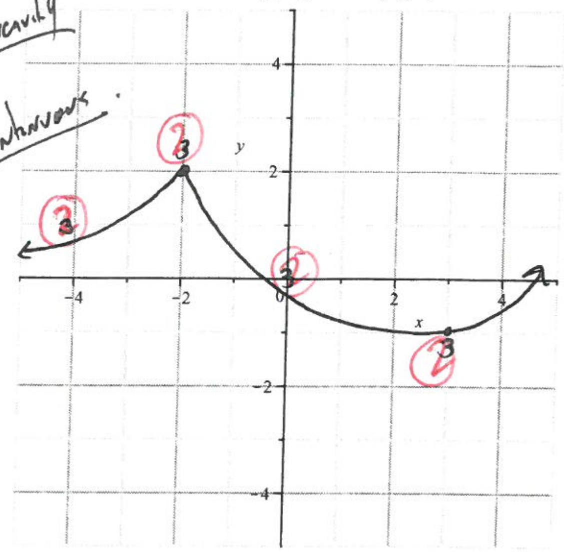
- $f''(x) > 0$ for $-\infty < x < -2$
- $f''(x) > 0$ for $-2 < x < \infty$

$f''>0$ | $f''>0$ ; $-2$

-3 for each MAX/MIN/concavity

-2 for continuous.

Possible graph of $f(x)$:

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7. Let $f(t) = t^3 - 3t^2 - 24t + 40$. Determine the critical points of $f$ and classify them as local minimums, local maximums or neither.

$f'(t) = 3t^2 - 6t - 24$

$= 3(t^2 - 2t - 8)$

$= 3(t-4)(t+2)$

6

$t = 4$, $t = -2$ critical points

1st deriv. test

$+$ | $-$ | $+$

$f'>0$ | $f'=0$ | $f'<0$ | $f'=0$ | $f'>0$ ; $-3$, $-2$, $0$, $4$, $5$

$x = -2$ is a local MAX

$x = 4$ is a local min.

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-6 for no test.

-2 for extra crit. pt

2nd deriv. test

$f''(t) = 6t - 6$

$f''(4) = 18 > 0 \rightarrow t = 4$ is local min

$f''(-2) = -18 < 0 \rightarrow t = -2$ is a local max.

$t = 4$ is a local min. 1

$t = -2$ is a local max 1



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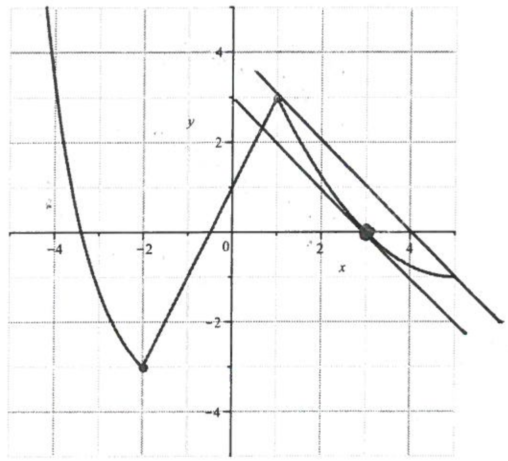
8. Recall the graph of $h(x)$ from Problem 1:

Graph of $h(x)$




(a) Approximate the value of $c$ such that

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$$h'(c) = \frac{h(5) - h(1)}{5 - 1}.$$

$\boxed{c \approx 3}$ 4

-4 for -1

(b) What is the name of the theorem which ensures that such a value of $c$ exists?

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Mean Value Theorem 2.

Local max slope of $e^{-x^2} = f$

$$f'(x) = -2xe^{-x^2} = g(x) \quad +1$$

$$f''(x) = -2e^{-x^2} + 4x^2e^{-x^2} = g'(x)$$

$$0 = 2e^{-x^2}\{2x^2 - 1\} \quad +2$$

max of g.

$$x = \pm 1/\sqrt{2}$$

$+ \quad \cancel{+} \quad - \quad +$

$-1 \quad \frac{-1}{\sqrt{2}} \quad 0 \quad \frac{1}{\sqrt{2}} \quad 1$

$+$

$+1$

occurs at $x = -1/\sqrt{2}$ is $f'(-1/\sqrt{2}) = -\frac{2}{\sqrt{2}} e^{1/2} = \boxed{-\sqrt{2}e^{1/2}}$

$+$

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