Exam III
Summer 2014

100/100

MATH 122B
Sections 4.2-4.7

Name: Key

You must *show work* and *simplify all answers* to receive full credit. If your work does not support your answer you will receive **no credit!**

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10 1. Determine (using calculus) the global maximum and global minimum of $f(x) = \frac{1}{3}(x^3 - 6x^2 + 9x + 1)$ on the interval $[0, 2]$. You must show all work – do not solve this problem graphically!

$$\left.\begin{aligned} f'(x) &= \tfrac{1}{3}(3x^2 - 12x + 9) \\ &= x^2 - 4x + 3 \\ &= (x-3)(x-1) \\ \text{Crit. pts: } &\underline{x = 1} \quad (x=3 \text{ is outside the domain!}). \end{aligned}\right\} 3$$

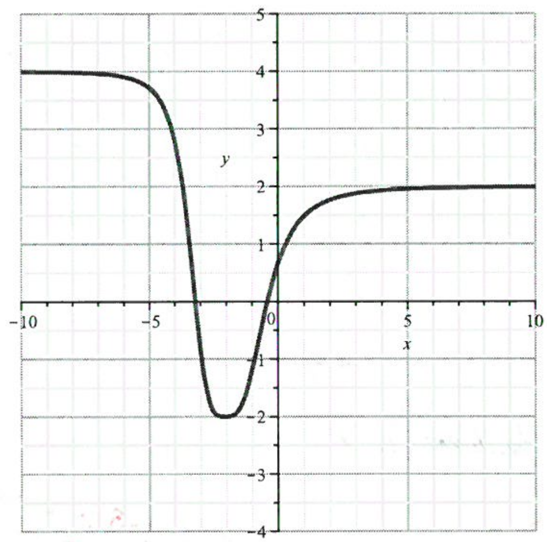
-2 if x=3 is used.

$$\left.\begin{aligned} f(1) &= \tfrac{1}{3}(1-6+9+1) = \tfrac{1}{3}(5) = 5/3 \\ f(0) &= 1/3 \\ f(2) &= \tfrac{1}{3}(8-24+18+1) = 1 \end{aligned}\right\} 5$$

Global MIN: $x = 0$	1
Global MAX: $x = 1$	1

delete.

16 2. Approximate the local and global extrema of the function whose graph is shown below on the following intervals: (If none exist, write "NONE")



(a) $[-4, \infty)$

4

Local Min(s): -2 ' (-2, -2) Global Min: -2 , (-2, -2)

Local Max(s): -4 ' (-4, 3) Global Max: -4 / (-4, 3)

2.5 ok

(b) $(-\infty, -4]$

4

Local Min(s): -4 , (-4, 3) Global Min: -4 , (-4, 3)

Local Max(s): None ' Global Max: None '

(c) $(-4, -2)$

4

Local Min(s): None ' Global Min: None '

Local Max(s): None ' Global Max: None '

(d) $(-\infty, \infty)$

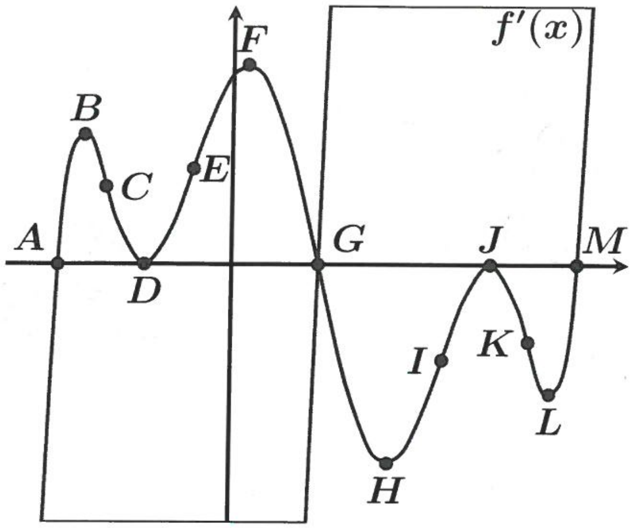
4

Local Min(s): ~~All~~ -2 ' (-2, -2) Global Min: ~~All~~ -2 , (-2, -2)

Local Max(s): None ' Global Max: None '

15

3. Consider the following graph of $f'(x)$:



Use the above graph to determine the following points:

(a) Critical point(s) of $f(x)$.
3

A, D, G, J, M

-1 for missg 2

(b) Local minimum(s) of $f(x)$.
3

A, M

-1 for missg 1.

-3 for D, H, L.

(c) Inflection point(s) of $f(x)$.
3

B, D, F, H, J, L

-3 Agm

-1 for missg 2

-2 for CEIKG ← inf. of f'.

(d) Local maximum(s) of $f'(x)$.
3

B, F, J

-1 per

(e) Local minimum(s) of $f''(x)$.
3

C, G, K

-1 per

-3 for HLD

-3 for BF.

-1 for MAX → EI.

Delete. Same flavor as 5.

12

4. Let

$$P(x) = R - 10(x+2)e^{-(x/10+1)}$$

denote the profit obtained from selling x items, where R is a fixed positive constant. How many items should be sold to maximize the profit?

$$P'(x) = -10(x+2)e^{-(\frac{x}{10}+1)}\left(-\frac{1}{10}\right) - 10e^{-(\frac{x}{10}+1)}$$

$$= (x+2)e^{-(\frac{x}{10}+1)} - 10e^{-(\frac{x}{10}+1)}$$

$$= e^{-(\frac{x}{10}+1)}\left((x+2)-10\right)$$

$$= e^{-(\frac{x}{10}+1)}(x-8)$$

2

Domain: $0 \le x < \infty$ 2

Crit. pt.: $x=8$.

— or —

$$P''(x) = \frac{-1}{10}e^{-(\frac{x}{10}+1)}(x-8) + e^{-(\frac{x}{10}+1)}$$

$$= e^{-(\frac{x}{10}+1)}\left\{1 - \frac{1}{10}(x-8)\right\}$$

$P''(8) > 0$

ccu ⇒ min

$$P(8) = R - 10(10)e^{-(8/10+1)} = R - \underbrace{100e^{-9/5}}_{16.5}$$

$$P(0) = R - 10(2)e^{-1} = R - \underbrace{20e^{-1}}_{7.3}$$

$$\lim_{x\to\infty} P(x) = R - 10(\infty)(0) \leftarrow \text{L'Hop.}$$

$$= \lim_{x\to\infty} R - \frac{10(x+2)}{e^{x/10+1}} = R - \frac{\infty}{\infty} \leftarrow \text{L'Hop.}$$

$$= \lim_{x\to\infty} R - \frac{10}{\frac{1}{10}e^{x/10+1}} = R - 0 = R.$$

6

-2 if forgot ∞.

-4 if say P(0)

-8 if say P(8) & no just

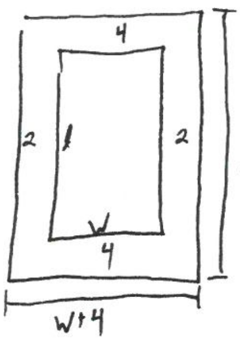
-4 for just wrong ↗↘ wrong

$$R > R - 100e^{-9/5} \quad \text{And} \quad R > R - 20e^{-1}$$

No Global MAX $\xrightarrow{2}$ Sell as much as possible.

12

5. You are designing a rectangular poster to contain N in^2 of printed text, where N is a fixed positive constant, with a 4 inch margin on the top and bottom, and a 2 inch margin on each side. What overall dimensions will minimize the amount of paper used?



$N = lw \Rightarrow \underbrace{l = N/w}_{2}$

$\boxed{\text{DOMAIN: } 0<w<\infty,\ 0<l<\infty.}$ 2

Give pts if N^2 instead

$$A = (l+8)(w+4)$$
$$= \left(\frac{N}{w}+8\right)(w+4)$$
$$= N + \frac{4N}{w} + 8w + 32$$
$$A' = -\frac{4N}{w^2} + 8 \qquad \frac{4N}{w^2} = 8 \qquad \frac{4N}{8} = w^2$$

3

Crit. pt: $w = \sqrt{N/2}$

$$A(\sqrt{N/2}) = N + \frac{4N}{\sqrt{N/2}} + 8\sqrt{N/2} + 32$$

$$\lim_{w\to 0} A(w) = \infty$$

$$\lim_{w\to\infty} A(w) = \infty$$

$w = \sqrt{N/2}$ gives global min.

$l = N/\sqrt{N/2} = \frac{N\sqrt{2}}{\sqrt{N}} = \sqrt{2N}$

4

$\boxed{\text{DIMENSIONS: } (\sqrt{N/2}+4) \times (\sqrt{2N}+8)}$ 1

no justify -4

10

6. Find a formula for the function of the form $y = ae^{-x} + bx$ with a global minimum at $(1, 2)$.

$2 = ae^{-1} + b$

$2 = \frac{a}{e} + b$ 3

$y' = -ae^{-x} + b$

$0 = -ae^{-1} + b$ 4

$\downarrow$

$\frac{a}{e} = b$

$2 = \frac{a}{e} + \frac{a}{e}$

$2 = 2\frac{a}{e}$

-1 not just?

$1 = \frac{a}{e}$ 2

$a = e$

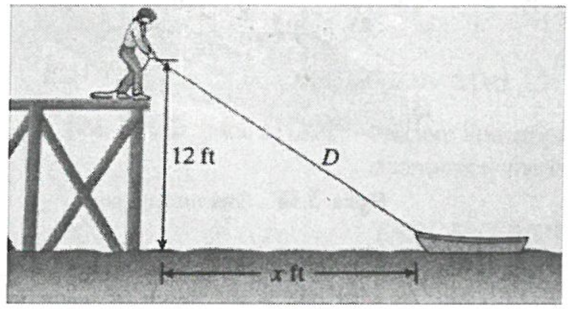
-1 No formula.

$b = 1$

$\boxed{y = e \cdot e^{-x} + x \quad \text{or} \quad y = e^{1-x} + x}$ 1

12

7. A person is standing at the end of a pier 12 ft. above the water and is pulling on a rope attached to a rowboat at the waterline at a rate of 6 ft. per minute. How fast is the boat moving in the water when it is 16 ft. from the pier?



$\frac{dD}{dt} = -6$

∘ what is $\frac{dx}{dt}$ when $x = 16$?

$x^2 + 12^2 = D^2 \rightarrow x^2 + 144 = D^2$ 2

$\downarrow$

$2x\frac{dx}{dt} = 2D\frac{dD}{dt}$

$\frac{dx}{dt} = \frac{D}{x}\frac{dD}{dt}$. 4

when $x = 16$,

$16^2 + 144 = D^2$

$256 + 144 = D^2$ 2

$400 = D^2$

$D = 20$.

-2 for switching x and D.

$\frac{dx}{dt} = \frac{20}{16}(-6)$

$= \frac{5}{4}(-6)$ 2

$= -\frac{15}{2} = \boxed{-7.5 \text{ ft/sec.}}$ 2

13

8. Evaluate the following (show any intermediate steps and justify the use of l'Hôpital's when needed):

(a) $\lim_{x \to 0} \frac{x - \cos x}{x^4}$ $= \frac{0-1}{0} = \frac{-1}{0} = \boxed{-\infty}$ -2 if L'Hôp

2

(b) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$ $= \frac{0-0}{0} = \frac{0}{0}$ ← CAN USE L'Hôp. -2 for no L'Hôp. verif.

4

$= \lim_{x\to 0} \frac{1-\cos x}{3x^2} = \frac{1-1}{0} = \frac{0}{0}$ ← CAN USE L'Hôp.

$= \lim_{x\to 0} \frac{\sin x}{6x} = \frac{0}{0}$ ← CAN USE L'Hôp.

$= \lim_{x\to 0} \frac{\cos x}{6} = \boxed{\frac{1}{6}}$

(c) $\lim_{x \to 0} \frac{x^2}{\cos x - \sin x}$ $= \frac{0}{1-0} = \frac{0}{1} = \boxed{0}$ -2 if L'Hôp.

2

(d) $\lim_{x \to \infty} \left(1 - \frac{2}{n}\right)^n$ $= (1-0)^\infty = 1^\infty$ ← use logs! 1pt.

5

rewrite! 1

$\lim_{n\to\infty} \ln\left(1-\frac{2}{n}\right)^n = \lim_{n\to\infty} n \ln\left(1-\frac{2}{n}\right) = \infty \cdot 0$

CAN USE L'Hôp! 1

$= \lim_{n\to\infty} \frac{\ln(1-2/n)}{1/n} = \frac{0}{0}$ ←

$= \lim_{n\to\infty} \frac{\frac{1}{1-2/n}\left(\frac{+2}{n^2}\right)}{(-1/n^2)} = \lim_{n\to\infty} \frac{-2}{1-2/n} = \frac{-2}{1} = -2 \Longrightarrow$ 1

$\lim_{n\to\infty} \ln\left(1-\frac{2}{n}\right)^n = -2$

$\Downarrow$

$\lim_{n\to\infty} \left(1-\frac{2}{n}\right)^n = \boxed{e^{-2}}$ 1