Chapter 2 Review MATH 122B

Some of these questions should start to sound repetitive (if some are repetitive know that you know what you are talking about, if every question sounds different you should start to get to the point where they do sound the same!).

1. Explain why the slope of a secant line can be interpreted as an average rate of change.

The slope of a secant line is the slope of a line connecting two points (which can be thought as the average rate of change between the two pts).

2. Explain why the slope of a tangent line can be interpreted as an instantaneous rate of change.

The slope of a tangent line is the slope of a secant line where the distance between the two points is going to zero.

3. Given the function f, what does f' represent?

The derivative of f.

(This is a fn)

4. Given a function f and a point a in its domain, what does $f'(a)$ represent? (Note, the answer should not identical to the answer above.)

The slope of the tangent line to $x=a$.

(This is always a numerical value)

5. Explain the relationships among the slope of a tangent line, the instantaneous rate of change, and the value of the derivative at a point.

See (2) + (4)

6. If f is differentiable at $x = a$, must f be continuous at $x = a$?

Yes!

7. If f is continuous at $x = a$, must f be differentiable at $x = a$?

No! Think $|x-a|$

8. If $f(4) = 3$ and $f'(4) = -5$ write the equation of the tangent line to $f(x)$ and $x = 4$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -5(x-4)$$
$$y = -5x + 23$$

9. Use the definition of the derivative to write the equation of the tangent line to the point stated below, on the below function:

(a) $(3,-14)$, $g(x) = -x^2 - 5$.

$$g'(x) = \lim_{h\to 0} \frac{-(x+h)^2 - 5 - (-x^2 - 5)}{h}$$
$$= \lim_{h\to 0} \frac{-(x^2 + 2xh + h^2) - 5 + x^2 + 5}{h}$$
$$= \lim_{h\to 0} \frac{-x^2 - 2xh - h^2 + x^2}{h}$$
$$= \lim_{h\to 0} \frac{-2xh - h^2}{h} = -\lim_{h\to 0} 2x - h$$
$$= -2x$$

$$g'(3) = -2(3) = -6$$
$$y + 14 = -6(x-3)$$
$$y = -6x + 4$$

(b) $(-1,4)$, $f(x) = \frac{4}{x^2}$.

$$f'(x) = \lim_{h\to 0} \frac{\frac{4}{(x+h)^2} - \frac{4}{x^2}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{4x^2 - 4(x+h)^2}{x^2(x+h)^2}}{h}$$
$$= \lim_{h\to 0} \frac{4x^2 - 4x^2 - 8xh - 4h^2}{x^2(x+h)^2 h}$$
$$= \lim_{h\to 0} \frac{-8xh - 4h^2}{x^2(x+h)^2 h} = \lim_{h\to 0} \frac{-8x - 4h}{x^2(x+h)^2}$$

$$g'(x) = \frac{-8x}{x^4} = \frac{-8}{x^3}$$
$$g'(-1) = 8$$
$$y - 4 = 8(x+1)$$
$$y = 8x + 12$$

10. Use the above equation of the tangent line to estimate the below points, and state whether this estimation is greater or less than the actual value. (Use the second derivative to explain why.)

(a) $g(3.5)$

$y = -6(3.5)+4 = -17$

$\left(\begin{array}{l} g(3.5) = -17.25 \\ \text{note } g(3.5) \neq y \end{array} \right)$

The estimation is greater because $g''(x) < 0$, or g is concave down.

(b) $f(-0.75)$

$y = 8(-0.75)+12$
$\quad = 6$

$\boxed{\begin{array}{l} f(-0.75) = \frac{4}{(-.75)^2} \\ \text{Note:} \qquad = 64/9 \approx 7.11 \end{array}}$

The estimation is less since $f''(-0.75) > 0$ or f is concave up.

11. Write the definition of the derivative of the following (do not solve):

(a) $f'(5)$ for $f(x) = e^{x+3}$

$$f'(5) = \lim_{h \to 0} \frac{e^{(5+h)+3} - e^{5+3}}{h}$$
$$= \lim_{h \to 0} \frac{e^{8+h} - e^{8}}{h}$$

(b) $g'(3)$ for $g(x) = \cos(x)$

$$g'(3) = \lim_{h \to 0} \frac{\cos(3+h) - \cos(3)}{h}$$

(c) $h'(2)$ for $h(x) = \ln(x) + x^2$

$$h'(2) = \lim_{h \to 0} \frac{\ln(2+h) + (2+h)^2 - \ln 2 - 2^2}{h}$$

12. Use the above expressions in Question 11 to estimate the derivative algebraically (know this means 'use small values of h').

Note we can't say $f'(5) =$, since this is an approximation!

(a) $h = 0.0001$ $\qquad$ $f'(5) \approx 2981.107$

(b) $h = 0.0001$ $\qquad$ $g'(3) \approx -0.141$

(c) $h = 0.0001$ $\qquad$ $h'(2) \approx 4.5$

13. Suppose the number of calculators sold, S, is a function of the calculators' price, $\$p$.

(a) What does $S(40)$ mean?

The number of calculators sold when the price is \$40.

(b) What does $S'(40)$ mean?

At the price of \$40 the number of calculators sold is increasing (if $S'(40)>0$) or decreasing (if $S'(40)<0$) by $|S'(40)|$. (Could also say... if \$1 is the "small amt" increasing price of calc from \$40 to \$41 calculator sales change by $S'(40)$.)

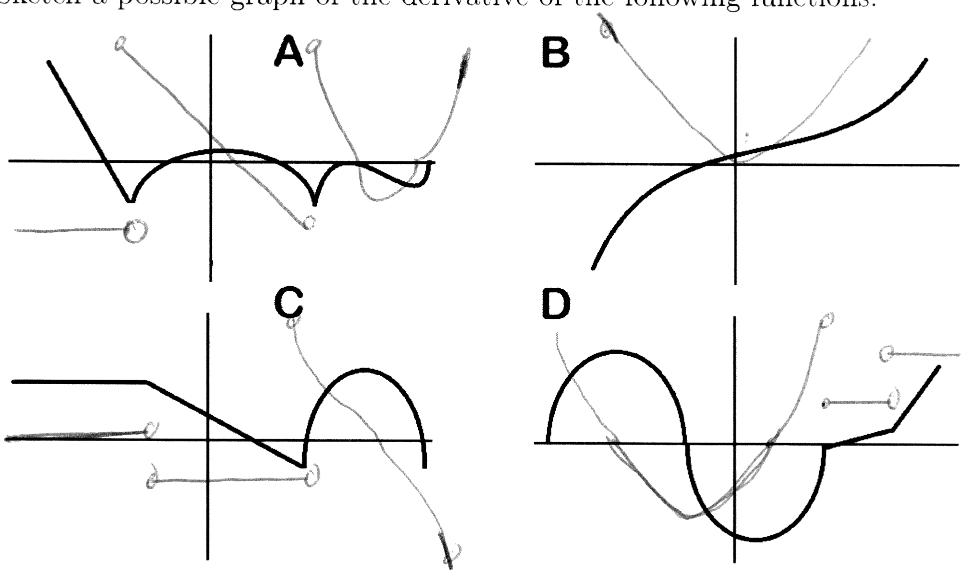
(c) What does $S^{-1}(40)$ mean?

The price in dollars is $S^{-1}(40)$ when 40 calculators are sold.

(d) What does $(S^{-1})'(40)$ mean? Units: $\frac{\Delta \$p}{\Delta \#\text{calc sold}}$

When the number of calculators sold changes from 40 to 41 the price increases (if $(S^{-1})'(40)>0$) or decreases (if $(S^{-1})'(40)<0$) by $|(S^{-1})'(40)|$.

14. Sketch a possible graph of the derivative of the following functions:



15. If you know that $f(x) > 0$ can you say anything about $f'(x)$ and $f''(x)$?

No! (nothing for sure)

16. If you know that $f(x) < 0$ can you say anything about $f'(x)$ and $f''(x)$?

No!

17. If you know that $f'(x) > 0$ can you say anything about $f(x)$ and $f''(x)$?

f - increasing

f'' - nothing

18. If you know that $f'(x) < 0$ can you say anything about $f(x)$ and $f''(x)$?

f - decreasing

f'' - nothing

19. If you know that $f''(x) > 0$ can you say anything about $f(x)$ and $f'(x)$?

f - concave up

f' - increasing

20. If you know that $f''(x) < 0$ can you say anything about $f(x)$ and $f'(x)$?

f - concave down

f' - decreasing

21. Determine whether the following statements are true or false, and given an explanation or counterexample.

(a) For linear functions, the slope of any secant line always equals the slope of any tangent line.

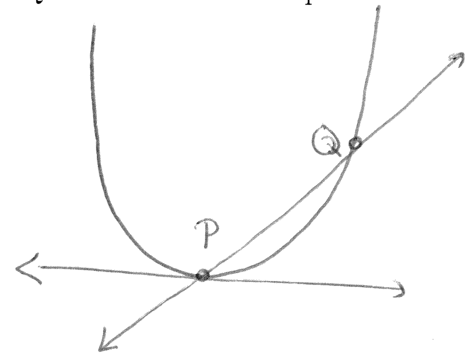
Yes! Let $y = mx + b = f(x)$ Slope of tan line $\underline{m}$

Slope of secant line: $\frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{(mx_1 + b) - (mx_2 + b)}{x_1 - x_2}$

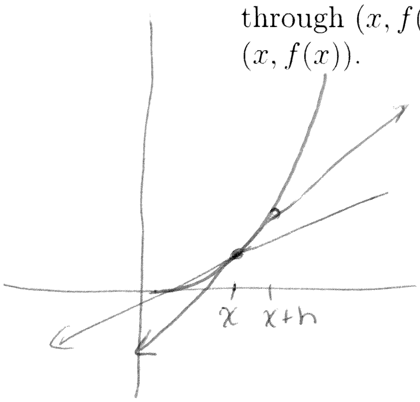
$= \frac{mx_1 - mx_2}{x_1 - x_2} = \frac{m(x_1 - x_2)}{x_1 - x_2} = m$

(b) The slope of the secant line passing through the points P and Q is less than the slope of the tangent line at P.

Not always



(c) Consider the graph of the parabola $f(x) = x^2$. For $x > 0$ and $h > 0$, the secant line through $(x, f(x))$ and $(x+h, f(x+h))$ always has a greater slope than the tangent line at $(x, f(x))$.



True. Graph, or algebraically

Slope of secant line: $\frac{f(x) - f(x+h)}{x - (x+h)}$

$= \frac{x^2 - (x^2+2xh+h^2)}{-h}$

$= \frac{-2xh - h^2}{-h} = 2x+h$

Slope of tan line: $f'(x) = 2x$

$2x < 2x+h$ when $h>0$.

22. Let

$$f(x) = \begin{cases} 2x^2 & \text{if } x \leq 1 \\ ax & \text{if } x > 1 \end{cases}$$

Determine a value of a (if possible) for which $f'(1)$ exists.

$$f'(x) = \begin{cases} 4x & \text{if } x < 1 \\ a & \text{if } x > 1 \end{cases}$$

Want $4x = a$ at $x = 1$

$4(1) = a$

So $a = 4$.

23. A magnetic field, B, is given as a function of the distance, r, from the center of a wire as follows:

$$B = \begin{cases} \frac{r}{r_0} B_0 & \text{for } r \leq r_0 \\ \frac{r_0}{r} B_0 & \text{if } r > r_0 \end{cases}$$

(a) Is B continuous at $r = r_0$? Explain.

Yes B evaluated at $r=r_0$ $\left(B\big|_{r=r_0} \right)$

is the same as $\underbrace{\lim_{r \to r_0^+} B}_{\text{Looking @ Bottom fn}} = \underbrace{\lim_{r \to r_0^-} B}_{\text{Looking @ top fn.}}$

(b) Is B differentiable at $r = r_0$? Explain.

$$B' = \begin{cases} B_0/r_0 & r < r_0 \\ \frac{-B_0 r_0}{r^2} & r > r_0 \end{cases}$$

Does $\lim_{r \to r_0^-} B' = \lim_{r \to r_0^+} B'$

no! $\frac{B_0}{r_0} \neq -\frac{B_0 r_0}{r_0^2}$

Not differentiable at $r=r_0$.