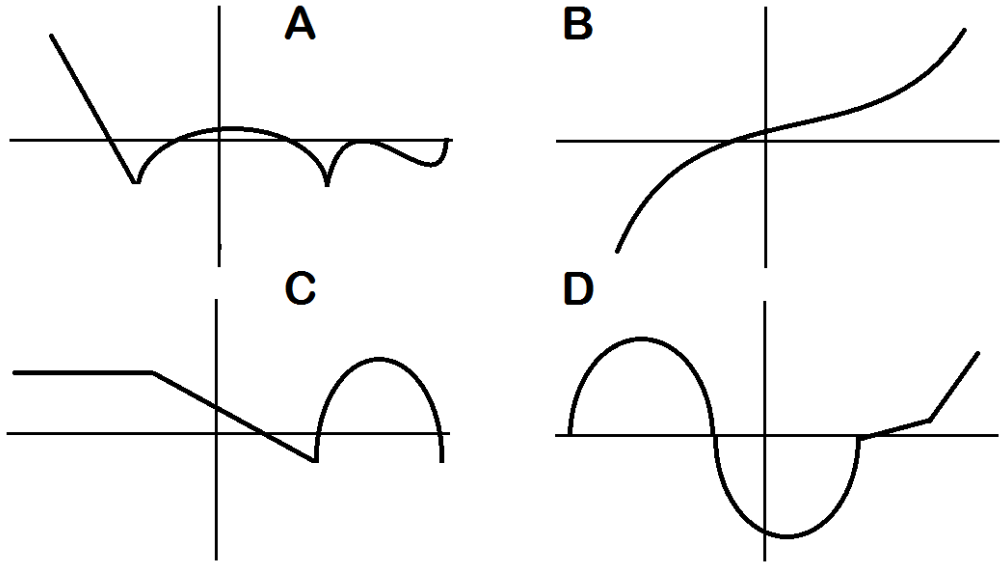
Chapter 2 Review MATH 122B

Some of these questions should start to sound repetitive (if some are repetitive know that you know what you are talking about, if every question sounds different you should start to get to the point where they do sound the same!).

1. Explain why the slope of a secant line can be interpreted as an average rate of change.
2. Explain why the slope of a tangent line can be interpreted as an instantaneous rate of change.
3. Given the function f, what does f' represent?
4. Given a function f and a point a in its domain, what does $f'(a)$ represent? (Note, the answer should not identical to the answer above.)
5. Explain the relationships among the slope of a tangent line, the instantaneous rate of change, and the value of the derivative at a point.
6. If f is differentiable at $x = a$, must f be continuous at $x = a$?
7. If f is continuous at $x = a$, must f be differentiable at $x = a$?
8. If $f(4) = 3$ and $f'(4) = -5$ write the equation of the tangent line to $f(x)$ and $x = 4$.
9. Use the definition of the derivative to write the equation of the tangent line to the point stated below, on the below function:
   (a) $(3, 4)$, $g(x) = -x^2 - 5$.
   (b) $(-1, 4)$, $f(x) = \dfrac{4}{x^2}$.
10. Use the above equation of the tangent line to estimate the below points, and state whether this estimation is greater or less than the actual value. (Use the second derivative to explain why.)
    (a) $g(3.5)$
    (b) $f(-0.75)$
11. Write the definition of the derivative of the following (do not solve):
    (a) $f'(5)$ for $f(x) = e^{x+3}$
    (b) $g'(3)$ for $g(x) = \cos(x)$
    (c) $h'(2)$ for $h(x) = \ln(x) + x^2$
12. Use the above expressions to estimate the derivative algebraically (know this means 'use small values of h').
    (a) $f'(5)$ for $f(x) = e^{x+3}$
    (b) $g'(3)$ for $g(x) = \cos(x)$
    (c) $h'(2)$ for $h(x) = \ln(x) + x^2$

13. Suppose the number of calculators sold, S, is a function of the calculators' price, $\$p$.
    (a) What does $S(40)$ mean?
    (b) What does $S'(40)$ mean?
    (c) What does $S^{-1}(40)$ mean?
    (d) What does $(S^{-1})'(40)$ mean?

14. Sketch a possible graph of the derivative of the following functions:



15. If you know that $f(x) > 0$ can you say anything about $f'(x)$ and $f''(x)$?

16. If you know that $f(x) < 0$ can you say anything about $f'(x)$ and $f''(x)$?

17. If you know that $f'(x) > 0$ can you say anything about $f(x)$ and $f''(x)$?

18. If you know that $f'(x) < 0$ can you say anything about $f(x)$ and $f''(x)$?

19. If you know that $f''(x) > 0$ can you say anything about $f(x)$ and $f'(x)$?

20. If you know that $f''(x) < 0$ can you say anything about $f(x)$ and $f'(x)$?

21. Determine whether the following statements are true or false, and given an explanation or counterexample.

    (a) For linear functions, the slope of any secant line always equals the slope of any tangent line.

    (b) The slope of the secant line passing through the points P and Q is less than the slope of the tangent line at P.

    (c) consider the graph of the parabola $f(x) = x^2$. For $x > 0$ and $h > 0$, the secant line through $(x, f(x))$ and $(x+h, f(x+h))$ always has a greater slope than the tangent line at $(x, f(x))$.

22. Let

$$f(x) = \begin{cases} 2x^2 & \text{if } x \leq 1 \\ ax & \text{if } x > 1 \end{cases}$$

    Determine a value of a (if possible) for which $f'(1)$ exists.

23. A magnetic field, B, is given as a function of the distance, r, from the center of a wire as follows:

$$B = \begin{cases} \dfrac{r}{r_0} B_0 & \text{for } r \leq r_0 \\ \dfrac{r_0}{r} B_0 & \text{if } r > r_0 \end{cases}$$

(a) Is B continuous at $r = r_0$? Explain.

(b) Is B differentiable at $r = r_0$? Explain.