Section 6.3

Examples

We want to discuss what the following questions are actually asking you to do:

1. Find the general solution to the differential equation

$$\frac{dw}{dx} = \sqrt{x} + 5.$$

This means, find the family of functions, $w(x)$, whose derivative equals $\sqrt{x} + 5$.

Answer: $w(x) = \frac{2}{3}x^{3/2} + 5x + C$.

2. Find the solution to the initial value problem

$$\frac{dx}{dt} = \csc^2 t + \frac{1}{t}, \qquad x(1) = 5.$$

This means, find the specific function $x(t)$ whose derivative equals $\csc^2 t + \frac{1}{t}$ and passes through the point $(1, 5)$.
First, find the general solution: $x(t) = -\cot t + \ln|t| + C$, then plug in the point to find the constant of integration.

Answer: $x(t) = -\cot t + \ln|t| + \cot 1 + 5$.

3. Find the position function for an object if its velocity is given by

$$\frac{ds}{dt} = -5t^2 + 3$$

and whose initial position is $s(0) = 10$.

This is the same as Problem 2. above, except it is in the form of an application.
First, find the general solution: $s(t) = -\frac{5}{3}t^3 + 3t + C$, then plug in the point to find the constant of integration.

Answer: $s(t) = -\frac{5}{3}t^3 + 3t + 10$.

4. Find the change in position of an object over the interval $1 \le t \le 3$ if the velocity function is given by

$$\frac{ds}{dt} = -5t^2 + 3.$$

This means, find the value of the definite integral

$$\int_1^3 \left(-5t^2 + 3\right) dt.$$

Notice the difference in what is being asked between this problem and Problem 3. above.

Answer: $-\frac{112}{3}$.