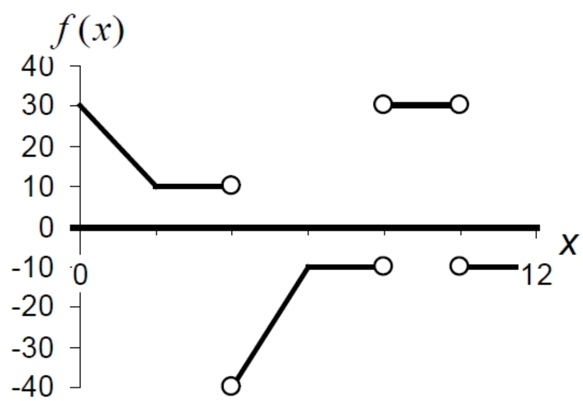
Section 6.1

Examples

The following are different ways to ask for the same thing:

- Given a derivative function, plot the original function.
- Given $g'(x)$, plot $g(x)$.
- Given $f(x)$, plot $F(x)$.
- Given a function, plot an antiderivative.
- Plot $\int_a^x f(t)dt$.

1. Use the graph of $f(x)$ below to sketch graphs of each of the following. Your graphs should have the appropriate characteristics (increasing/decreasin/constant, concave up/down, etc.) and coordinates. Assume $F'(x) = f(x)$.



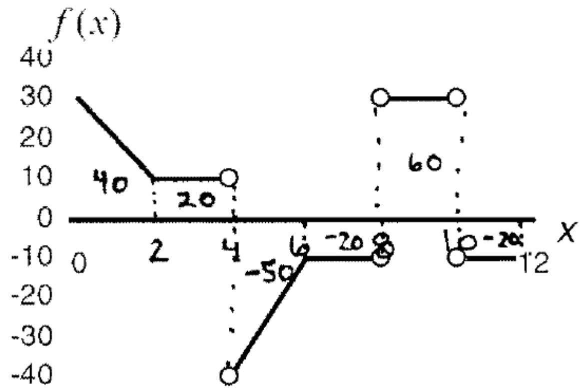
   (a) $F(x)$ where $F(0) = 0$
   (b) $F(x)$ where $F(0) = 10$
   (c) $F(x)$ where $F(4) = 0$
   (d) $F(x)$ where $F(4) = 20$

2. Suppose $G(x)$ is a function such that $G(0) = 5$ and $G'(x) = e^{-0.2x}\cos x$.

   (a) Find the critical point of $G(x)$ on the interval $0 \leq x \leq 4$ and classify it as either a local maximum or minimum. Give an exact answer.
   (b) Estimate the y-coordinate of your answer in part (a).

Solutions.

1. Use the graph of $f(x)$ below to sketch graphs of each of the following. Your graphs should have the appropriate characteristics (increasing/decreasin/constant, concave up/down, etc.) and coordinates. Assume $F'(x) = f(x)$.

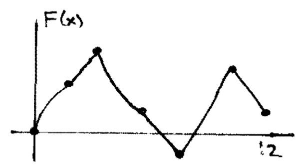


   (a) $F(x)$ where $F(0) = 0$

   First, we note that $F(x) = \int_0^x f(t)dt$. Using this, and the above graph we can create the following table of values:

x	0	2	4	6	8	10	12
$F(x)$	0	40	60	10	-10	50	30

   From our table we can now sketch a graph of $F(x)$:



   (b) $F(x)$ where $F(0) = 10$

$$\begin{aligned} \int_0^x f(t)dt &= F(x) - F(0) \\ &= F(x) - 10, \end{aligned}$$

   so we see that $F(x) = \int_0^x f(t)dt + 10$. Thus, the graph will look identical to the graph from part (a), except it will be shifted up 10 units.

   (c) $F(x)$ where $F(4) = 0$

$$\begin{aligned} \int_4^x f(t)dt &= F(x) - F(4) \\ &= F(x) - 0 \\ &= F(x), \end{aligned}$$

   so we have that $F(x) = \int_4^x f(t)dt$. Using the properties of definite integrals, we can rewrite this as:

$$F(x) = \int_4^x f(t)dt = \int_0^x f(t)dt - \int_0^4 f(t)dt.$$

From our table in part (a) we know that $\int_0^4 f(t)dt = 60$, thus, $F(x) = \int_0^x f(t)dt - 60$. Thus, the graph will look identical to the graph from part (a), except it will be shifted down 60 units.

(d) $F(x)$ where $F(4) = 20$

$$\begin{aligned} \int_4^x f(t)dt &= F(x) - F(4) \\ &= F(x) - 20 \end{aligned}$$

so we have that $F(x) = \int_4^x f(t)dt + 20$. Thus, the graph will look identical to the graph from part (c), except shifted up by 20 units. (Or equivalently, it will look identical to the graph from part (a), except shifted down 40 units.)

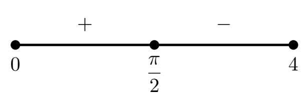
2. Suppose $G(x)$ is a function such that $G(0) = 5$ and $G'(x) = e^{-0.2x}\cos x$.

(a) Find the critical point of $G(x)$ on the interval $0 \leq x \leq 4$ and classify it as either a local maximum or minimum. Give an exact answer.

The critical point of $G(x)$ can be found by solving $G'(x) = 0$ (as usual). Solving

$$0 = e^{-0.2x}\cos x$$

on the interval $[0, 4]$, we find $x = \frac{\pi}{2}$.



The critical point $x = \frac{\pi}{2}$ is a local max. by the First Derivative Test.

(b) Estimate the y-coordinate of your answer in part (a).

$$\begin{aligned} \int_0^x e^{-0.2t}\cos t dt &= G(x) - G(0) \\ &= G(x) - 5, \end{aligned}$$

so we find that $G(x) = \int_0^x e^{-0.2t}\cos t dt_5$. Using approximation techniques (Riemann sums/calculator/etc.) we can find

$$\begin{aligned} G\left(\frac{\pi}{2}\right) &= \int_0^{\pi/2} e^{-0.2t}\cos t dt + 5 \\ &\approx 0.9103 + 5 \\ &= 5.9103 \end{aligned}$$

(Note, in Calculus II you will develop a method for computing this integral exactly!)