

Section 5.4

Examples

1. For $a < c < b$ suppose we know:

$$\int_a^b f(x)dx = 10, \quad \int_a^b (f(x))^2 dx = 15, \quad \int_a^c f(x)dx = 4.$$

Find values for each of the following:

(a) $\int_a^b 6f(x)dx$

(b) $\int_a^b (6 + f(x)) dx$

(c) $\left(\int_a^b f(x)dx\right)^2$

(d) $\int_b^a f(x)dx$

(e) $\int_b^b f(x)dx$

(f) $\int_c^b f(t)dt$

(g) $\int_{a-3}^{b-3} f(x+3)dx$

2. Set up definite integrals for the following areas. Do not evaluate the integrals.

(a) The area under $f(x) = e^{-2x}$ over the interval $[-2, 5]$.

(b) The area below $g(x) = -3x^2 + 15$ and between the function's x -intercepts.

(c) The area between $f(\theta) = \sin \theta$ and $g(\theta) = \cos \theta$ for $0 \leq \theta \leq A$, where A is the first intersection of the two graphs.

Solutions.

1. For $a < c < b$ suppose we know:

$$\int_a^b f(x)dx = 10, \quad \int_a^b (f(x))^2 dx = 15, \quad \int_a^c f(x)dx = 4.$$

Find values for each of the following:

(a) $\int_a^b 6f(x)dx = 60$

(b) $\int_a^b (6 + f(x)) dx = 6(b - a) + 10$

(c) $\left(\int_a^b f(x)dx \right)^2 = 100$

(d) $\int_b^a f(x)dx = -10$

(e) $\int_b^b f(x)dx = 0$

(f) $\int_c^b f(t)dt = 6$

(g) $\int_{a-3}^{b-3} f(x + 3)dx = 10$

2. Set up definite integrals for the following areas. Do not evaluate the integrals.

- (a) The area under $f(x) = e^{-2x}$ over the interval $[-2, 5]$.

$$\int_{-2}^5 e^{-2x} dx$$

- (b) The area below $g(x) = -3x^2 + 15$ and between the function's x -intercepts.

$$\int_{-\sqrt{5}}^{\sqrt{5}} (-3x^2 + 15) dx$$

- (c) The area between $f(\theta) = \sin \theta$ and $g(\theta) = \cos \theta$ for $0 \leq \theta \leq A$, where A is the first intersection of the two graphs.

$$\int_0^{\pi/4} (\cos \theta - \sin \theta) d\theta$$