Section 5.3

Examples

1. A log begins to lose weight after it is cut due to drying. If $\int_0^{100} w(t)dt$ represents the total weight loss (in pounds) due to drying over the first 100 days, what does $w(t)$ represent?

2. Salinity (salt concentration in gm/liter per cm) in sea water varies according to depth from the surface of the water (cm). If $s(x)$ is the rate of change of salinity x cm from the surface, write a definite integral to represent the change in salinity over the first 5 cm below the surface of the water.

3. Let $C(n)$ be a city's cost, in millions of dollars, for plowing roads when n inches of snow have fallen. Give a practical interpretation of each of the following:

   (a) $C'(15) = 0.7$

   (b) $\int_{12}^{24} C'(n)dn$

4. Water is flowing into a reservoir at the rate of $r(t)$ gallons per hour.

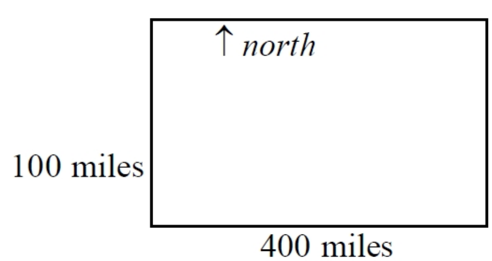
   (a) Give a practical interpretation of $\int_0^4 r(t)dt$.

   (b) Estimate the total number of gallons of water that would flow into the reservoir during the first day if $r(t) = \frac{100}{t^2+1}$. Use the left-hand sum and right-hand sum with $n = 48$.

   (c) Which estimate in part (b) is an overestimate and which is an underestimate for the total number of gallons? How do you know?

   (d) How often is the function evaluated in the calculations in part (b)? How often would it need to be calculated for the difference between the left- and right-hand sums to be no more than 1 gallon?

5. The density of trees in a particular rectangular plot of forest is given by $\delta(x)$, measured in hundreds of trees per square mile. If x represents the number of miles from the eastern edge, set up a definite integral to represent the total number of trees in this plot.



Solutions.

1. A log begins to lose weight after it is cut due to drying. If $\int_0^{100} w(t)dt$ represents the total weight loss (in pounds) due to drying over the first 100 days, what does $w(t)$ represent?

   The rate in pounds per day that the log loses weight.

2. Salinity (salt concentration in gm/liter per cm) in sea water varies according to depth from the surface of the water (cm). If $s(x)$ is the rate of change of salinity x cm from the surface, write a definite integral to represent the change in salinity over the first 5 cm below the surface of the water.

$$\int_0^5 s(x)dx$$

3. Let $C(n)$ be a city's cost, in millions of dollars, for plowing roads when n inches of snow have fallen. Give a practical interpretation of each of the following:

   (a) $C'(15) = 0.7$

   It will cost the city approximately an additional $700,000 for plowing roads if 16 inches of snow fall instead of 15 inches of snow.

   (b) $\int_{12}^{24} C'(n)dn$

   This is the difference in cost to the city (in millions of dollars) for plowing roads if 24 inches of snow falls instead of 12 inches of snow.

4. Water is flowing into a reservoir at the rate of $r(t)$ gallons per hour.

   (a) Give a practical interpretation of $\int_0^4 r(t)dt$.

   This is the total number of gallons flowing into the reservoir in the first 4 hours.

   (b) Estimate the total number of gallons of water that would flow into the reservoir during the first day if $r(t) = \frac{100}{t^2+1}$. Use the left-hand sum and right-hand sum with $n = 48$.

   Left-hand sum: 177.87 gallons
   Right-hand sum: 127.96 gallons

   (c) Which estimate in part (b) is an overestimate and which is an underestimate for the total number of gallons? How do you know?

   The left sum is an overestimate because the graph of $r(t)$ decreases on that interval. The right hand sum is an underestimate.

(d) How often is the function evaluated in the calculations in part (b)? How often would it need to be calculated for the difference between the left- and right-hand sums to be no more than 1 gallon?

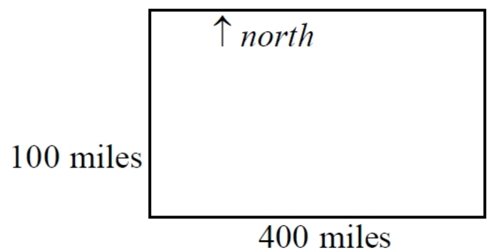
Since $\frac{24-0}{48-0} = 0.5$ hours, the rate function is evaluated every 30 minutes in part (b)'s calculation.

We want to find Δt for which, $|r(24) - r(0)| \cdot \Delta t \leq 1$. Thus,

$$\Delta t \leq \frac{1}{\left|\frac{100}{24^2+1} - \frac{100}{0+1}\right|} \approx 0.0100,$$

so we need to evaluate the rate function every 0.6 minutes.

5. The density of trees in a particular rectangular plot of forest is given by $\delta(x)$, measured in hundreds of trees per square mile. If x represents the number of miles from the eastern edge, set up a definite integral to represent the total number of trees in this plot.



$$\int_0^{400} \delta(x) \cdot 100dx \text{ hundreds of trees}$$