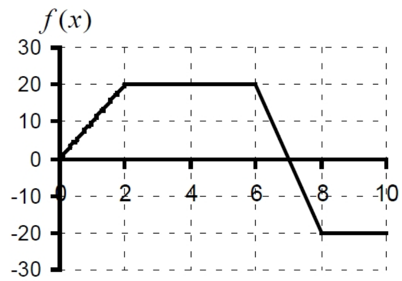
Section 5.2
Examples

1. The graph of $f(x)$ is shown below.



Use geometry to find each of the following:

(a) $\int_0^6 f(x)dx$

(b) $\int_6^8 f(x)dx$

(c) $\int_0^{10} f(x)dx$

(d) $\int_0^{10} |f(x)|\, dx$

2. Sketch a graph of

$$g(t) = \begin{cases} \sqrt{9-t^2} & -3 \le t \le 3 \\ 4t - 12 & 3 < t \le 5 \end{cases}$$

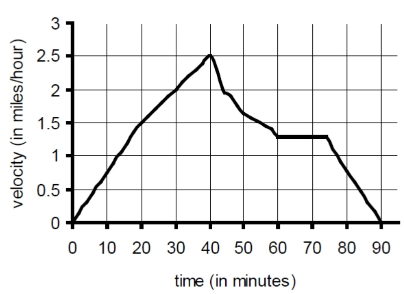
Use geometry to find $\int_{-3}^{5} g(t)dt$.

3. The velocity of an object is given by

$$v(t) = e^{0.1t} \sin\left(\frac{\pi}{6}t\right) \text{ for } 1 \le t \le 5.$$

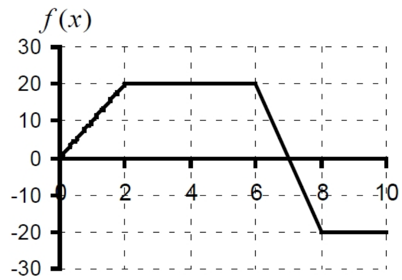
Use the left-hand sum and the right-hand sum with $n = 20$ to estimate $\int_1^5 v(t)dt$. Is this total distance traveled, or net displacement? How do you know?

4. Use the graph below to estimate total distance traveled during the first 90 minutes. Watch your units!



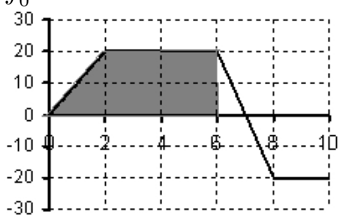
Solutions.

1. The graph of $f(x)$ is shown below.

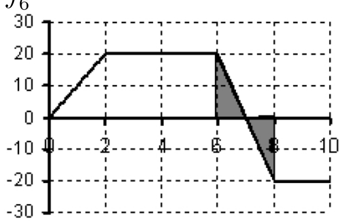


Use geometry to find each of the following:

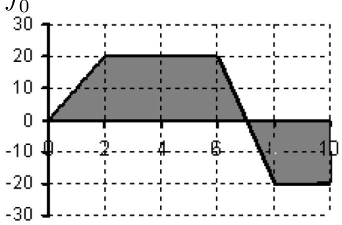
(a) $\int_0^6 f(x)dx = 100$



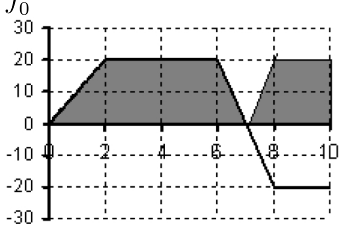
(b) $\int_6^8 f(x)dx = 0$



(c) $\int_0^{10} f(x)dx = 60$



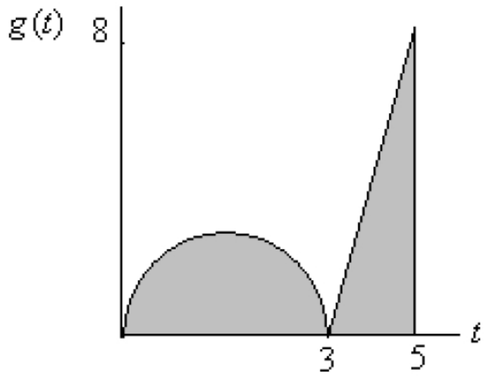
(d) $\int_0^{10} |f(x)|\, dx = 160$



2. Sketch a graph of

$$g(t) = \begin{cases} \sqrt{9-t^2} & -3 \le t \le 3 \\ 4t - 12 & 3 < t \le 5 \end{cases}$$

Use geometry to find $\int_{-3}^{5} g(t)dt$.



$$\int_{-3}^{5} g(t)dt = \frac{1}{2}\pi(3)^2 + \frac{1}{2}(2)(8) = \frac{9\pi}{2} + 8.$$

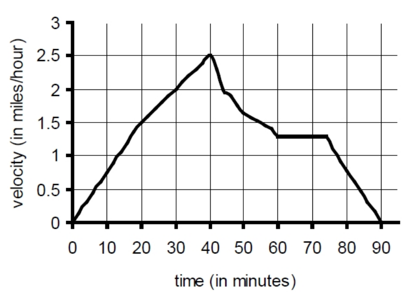
3. The velocity of an object is given by

$$v(t) = e^{0.1t}\sin\left(\frac{\pi}{6}t\right) \text{ for } 1 \le t \le 5.$$

Use the left-hand sum and the right-hand sum with $n = 20$ to estimate $\int_{1}^{5} v(t)dt$. Is this total distance traveled, or net displacement? How do you know?

Left-hand Sum: 4.4590
Right-hand Sum: 4.5134
This integral represents both distance traveled and net displacement because the graph is positive on the given interval!

4. Use the graph below to estimate total distance traveled during the first 90 minutes. Watch your units!



$$\text{Distance} = \frac{\text{miles}}{\text{hours}} \cdot \text{minutes} \cdot \frac{1}{60} = \text{miles}$$

Each "box" has an area equal to 5 and approximately 23.5 boxes fall under the graph. Thus,

$$\text{Distance} \approx (23.5)(5)\left(\frac{1}{60}\right) \approx 1.958 \text{ miles.}$$