

Section 5.1

How to Estimate

How should we find estimates for

$$\int_a^b f(x)dx?$$

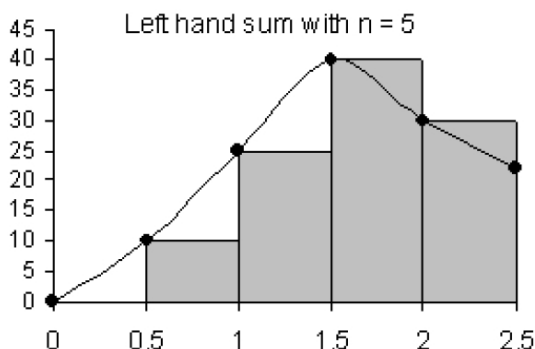
Tables:

If given a table of values, we are restricted to the size of Δx that is given by the x -values in the table. Pay close attention to what is being asked. Do we need a specific rule (left-hand versus right-hand) or a specific estimate (underestimate or overestimate)?

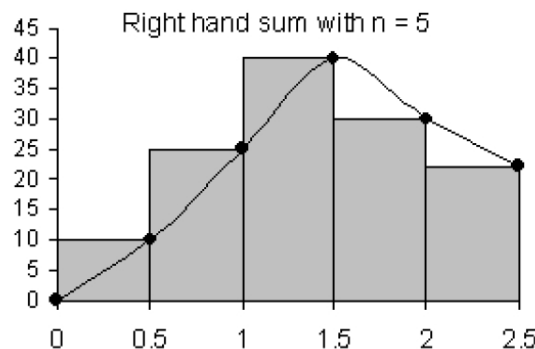
Example: You are waiting at an intersection when the light turns green. Your car's velocity as a function from the time the light turns green is given below. Estimate the distance you traveled during the first 2.5 seconds.

| | | | | | | |
|-------------------|-----|-----|-----|-----|-----|-----|
| Time (sec) | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
| Velocity (ft/sec) | 0 | 10 | 25 | 40 | 30 | 22 |

These illustrations use the rules without regard to under- or overestimates:

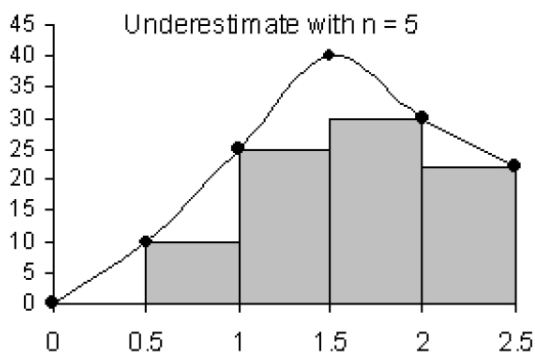


$$0(0.5) + 10(0.5) + 25(0.5) + 40(0.5) + 30(0.5) = 52.5$$

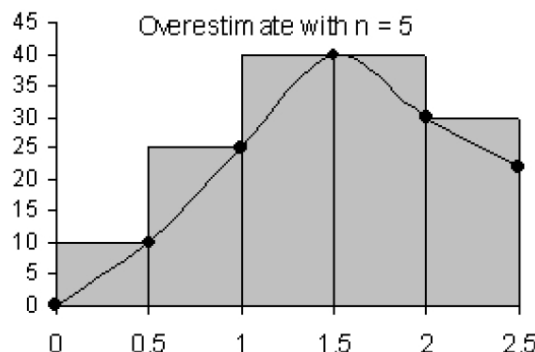


$$10(0.5) + 25(0.5) + 40(0.5) + 30(0.5) + 22(0.5) = 63.5$$

These illustrations use under- or overestimates without regard for the rules:



$$0(0.5) + 10(0.5) + 25(0.5) + 30(0.5) + 22(0.5) = 43.5$$

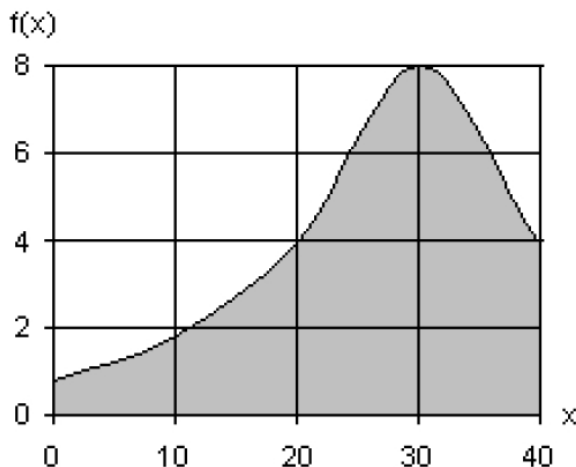


$$10(0.5) + 25(0.5) + 40(0.5) + 40(0.5) + 30(0.5) = 72.5$$

Graphs:

If given a graph with a scale on the axes, count boxes and fractions of boxes, then multiply by the area of one box. Estimate fractions of boxes as closely as possible. Use left and right sums only if asked. Use symmetry if possible.

Example: Estimate the area under the graph of $f(x)$ over the interval $[0, 40]$:



The area of each box is 20 square units. Counting parts of boxes, there are approximately 8 boxes under the graph. We can then say

$$\int_0^{40} f(x)dx \approx 8 \cdot 20 = 160 \text{ square units}$$

Equations:

If given a formula for $f(x)$ and a small value for n , we could approach the problem as if we were given a table. (If n is relatively large, you should not try to do this by hand – use technology!)

- Use the integration utility at: <http://www.zweigmedia.com/RealWorld/integral/integral.html>. This allows you to control the type of rule you use and the number of subintervals.
- The built-in numerical integration feature found in your graphing calculator does not allow you to choose a specific estimate. If the left or right sum is specifically asked for, your calculator will not produce that value! If there are no instructions on what type of estimate you need, then you can use your calculator.
- The LR Sums program posted at: <http://math.arizona.edu/~krawczyk/calcul.html> can be loaded into your calculator as a program. If you plan to continue into Calculus II, the ALLSUMS program includes the other rules covered in that course along with the left and right rules.

On an exam you would only be asked to estimate for a small n and need to do the work by hand!

Example: Estimate the area under $f(x) = e^{\cos x}$ over the interval $[1, 5]$ using a left-hand sum with $n = 50$.

$$\int_1^5 e^{\cos x} dx \approx 2.85884 \text{ using the left-hand sum with } n = 50.$$