Section 4.4

Example

For positive A and B, the force between two atoms is given by

$$F(r) = \frac{B}{r^3} - \frac{A}{r^2}$$

where $r > 0$ is the distance between the two atoms.

1. Find all intercepts and asymptotes of $F(r)$.

2. Find all critical points of $F(r)$ and classify them as local maxima or minima.

3. Find all inflection points of $F(r)$.

Solution.

For positive A and B, the force between two atoms is given by

$$F(r) = \frac{B}{r^3} - \frac{A}{r^2}$$

where $r > 0$ is the distance between the two atoms.

1. Find all intercepts and asymptotes of $F(r)$.

   Horizontal Intercept:

   $$\begin{aligned} \frac{B}{r^3} - \frac{A}{r^2} &= 0 \\ \frac{B - Ar}{r^3} &= 0 \\ B - Ar &= 0 \\ r &= \frac{B}{A}. \end{aligned}$$

   Horizontal intercept: $\left(\frac{B}{A}, 0\right)$.

   Vertical Intercept:
   None.

   Horizontal Asymptote:

   $$\lim_{r \to \infty} \frac{B - Ar}{r^3} = 0$$

   Horizontal asymptote: $y = 0$.

   Vertical Asymptote:

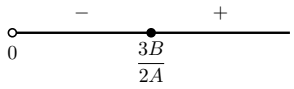
   $$r^3 = 0$$

   Vertical asymptote: $r = 0$.

2. Find all critical points of $F(r)$ and classify them as local maxima or minima.

   $$F'(r) = -\frac{3B}{r^4} + \frac{2A}{r^3} = \frac{-3B + 2Ar}{r^4}.$$

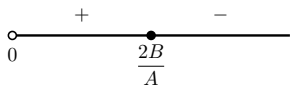
   Critical point: $r = \frac{3B}{2A}$

   Local minimum: $r = \frac{3B}{2A}$.

3. Find all inflection points of $F(r)$.

   $$F''(r) = \frac{12B}{r^5} - \frac{6A}{r^4} = \frac{12B - 6Ar}{r^5}.$$



Inflection point: $r = \dfrac{2B}{A}$.