Section 3.8

Examples

Definitions:

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}$$

1. Simplify the expression $\sinh(\ln(2t))$.

2. Using the definition, find $\lim_{t\to\infty} \frac{1}{\cosh(3t)}$.

3. When a particular cable hangs between two poles located at $x = -a$ and $x = a$, the shape is called a **catenary** and is given by $f(x) = a\cosh\left(\frac{x}{a}\right)$. Find an expression for the sag (the difference between the highest and lowest points of the cable).

4. If you fall through a medium so that resistance is proportional to the square of your velocity, then the distance you fall as a function of time is given by $x(t) = k\ln(\cosh(ct))$.

   (a) Find a formula for your velocity.

   (b) What is the limiting value of your velocity?

Solutions.

Definitions:

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}$$

1. Simplify the expression $\sinh(\ln(2t))$.

$$\sinh(\ln(2t)) = \frac{e^{\ln(2t)} - e^{-\ln(2t)}}{2} = \frac{2t - \frac{1}{2t}}{2} = \frac{4t^2 - 1}{4t}.$$

2. Using the definition, find $\lim_{t\to\infty} \frac{1}{\cosh(3t)}$.

$$\lim_{t\to\infty} \frac{1}{\cosh(3t)} = \lim_{t\to\infty} \frac{2}{e^{3t} + e^{-3t}} = 0.$$

3. When a particular cable hangs between two poles located at $x = -a$ and $x = a$, the shape is called a **catenary** and is given by $f(x) = a\cosh\left(\frac{x}{a}\right)$. Find an expression for the sag (the difference between the highest and lowest points of the cable).

$$f(a) - f(0) = a\cosh(1) - a\cosh(0) = a\left(\cosh(1) - 1\right).$$

4. If you fall through a medium so that resistance is proportional to the square of your velocity, then the distance you fall as a function of time is given by $x(t) = k\ln(\cosh(ct))$.

   (a) Find a formula for your velocity.

   $$x'(t) = k\frac{1}{\cosh(ct)}\left(\sinh(ct)\right)c = ck\tanh(ct).$$

   (b) What is the limiting value of your velocity?

   $$\lim_{t\to\infty} ck\tanh(ct) = ck\lim_{t\to\infty} \frac{e^{ct} - e^{-ct}}{e^{ct} + e^{-ct}} = ck\lim_{t\to\infty} \frac{1 - e^{-2ct}}{1 + e^{-2ct}} = ck.$$