Section 3.5

Examples

1. Find the first derivative for each function.
   (a) $f(\theta) = \dfrac{1}{\sin(3\theta)}$
   (b) $h(t) = \sec^3(t)$
   (c) $p(z) = z^3 \cos(z^2)$
   (d) $g(x) = \tan(\sin(x))$
   (e) $p(v) = 3^{\cos(v)}$
   (f) $r(y) = \sqrt[3]{y + \cot^2(3y)}$
   (g) $p(\theta) = \frac{\cos(\theta)}{1+\sin(\theta)}$

2. Determine if the graph of $f(x) = \sin(x^4)$ is increasing or decreasing at $x = 10$. Then determine whether it is concave up or concave down at $x = 10$.

Solutions.

1. Find the first derivative for each function.

(a) $f(\theta) = \dfrac{1}{\sin(3\theta)}$

$$\begin{aligned} f(\theta) &= \csc(3\theta) \\ f'(\theta) &= -\csc(3\theta)\cot(3\theta)\cdot 3 \\ &= -3\csc(3\theta)\cot(3\theta). \end{aligned}$$

(b) $h(t) = \sec^3(t)$

$$\begin{aligned} h'(t) &= 3\sec^2(t)\cdot\sec(t)\tan(t) \\ &= 3\sec^3(t)\tan(t). \end{aligned}$$

(c) $p(z) = z^3\cos(z^2)$

$$\begin{aligned} p'(z) &= z^3\cdot(-\sin(z^2)\cdot 2z) + \cos(z^2)\cdot 3z^2 \\ &= -2z^4\sin(z^2) + 3z^2\cos(z^2). \end{aligned}$$

(d) $g(x) = \tan(\sin(x))$

$$g'(x) = \sec^2(\sin(x))\cdot\cos(x).$$

(e) $p(v) = 3^{\cos(v)}$

$$\begin{aligned} p'(v) &= \ln(3)\cdot 3^{\cos(v)}(-\sin(v)) \\ &= -\ln(3)\sin(v)3^{\cos(v)}. \end{aligned}$$

(f) $r(y) = \sqrt[3]{y + \cot^2(3y)}$

$$\begin{aligned} r(y) &= \left(y + (\cot(3y))^2\right)^{1/3} \\ r'(y) &= \frac{1}{3}\left(y + (\cot(3y))^2\right)^{-2/3}\left(1 + 2\cot(3y)\cdot(-\csc^2(3y)\cdot 3)\right) \\ &= \frac{1 - 6\cot(3y)\csc^2(3y)}{3\left(+\cot^2(3y)\right)^{2/3}}. \end{aligned}$$

(g) $p(\theta) = \frac{\cos(\theta)}{1+\sin(\theta)}$

$$\begin{aligned} p'(\theta) &= \frac{(1+\sin(\theta))(-\sin(\theta)) - \cos(\theta)\cos(\theta)}{(1+\sin(\theta))^2} \\ &= \frac{-\sin(\theta) - 1}{(1+\sin(\theta))^2} \\ &= \frac{1}{1+\sin(\theta)}. \end{aligned}$$

2. Determine if the graph of $f(x) = \sin(x^4)$ is increasing or decreasing at $x = 10$. Then determine whether it is concave up or concave down at $x = 10$.

$$\begin{aligned} f'(x) &= \cos(x^4) \cdot 4x^3 \\ &= 4x^3 \cos(x^4) \\ f'(10) &= 4(10)^3 \cos(10^4) < 0. \end{aligned}$$

Since $f'(10) < 0$, f is decreasing at $x = 10$.

$$\begin{aligned} f''(x) &= 4x^3 \cdot (-\sin(x^4)) \cdot 4x^3 + \cos(x^4) \cdot 12x^2 \\ &= -16x^6 \sin(x^4) + 12x^2 \cos(x^4) \\ f''(10) &= -16(10)^6 \sin(10^4) + 12(10)^2 \cos(10)^4 > 0. \end{aligned}$$

Since $f''(10) > 0$, f is concave up at $x = 10$.