Section 3.4

Examples

1. Find the first derivative for each function.
   (a) $f(x) = (4x^3 - 3x + 7)^8$
   (b) $g(y) = \dfrac{1}{5(1-2y^2)}$
   (c) $P(t) = 5.2e^{-0.4t}$
   (d) $m(z) = \dfrac{-6}{\sqrt[5]{(z^2+4)^3}}$
   (e) $n(t) = \ln\left(e^{7t+1}\right)$
   (f) $y = 5^{\sqrt{x}}$
   (g) $f(x) = (ax+b)^3(3-x)$
   (h) $h(p) = \left(\dfrac{2p-3}{5-p}\right)^7$

2. Find the first derivative in each. In these problems you need to simplify your answer in a special way. The solutions to these are posted in a video in D2L under Section 3.4.
   (a) $g(x) = (2x+1)^4(4x-3)^5$
   (b) $p(v) = \dfrac{5v}{(v^2+1)^3}$
   (c) $r(y) = \dfrac{4y^2}{\sqrt[3]{2-y^2}}$

3. Find each of the indicated derivatives for the function below. The notation in each part tells you which letter represents the independent variable (the remaining letters will be parameters).

$$y = \frac{m}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}.$$

   (a) $\dfrac{dy}{dm}$
   (b) $\dfrac{dy}{dv}$
   (c) $\dfrac{dy}{dc}$

Solutions.

1. Find the first derivative for each function.

   (a) $f(x) = (4x^3 - 3x + 7)^8$

$$\begin{aligned} f'(x) &= 8(4x^3 - 3x + 7)^7 \cdot (12x^2 - 3) \\ &= 8(12x^2 - 3)(4x^3 - 3x + 7)^7. \end{aligned}$$

   (b) $g(y) = \dfrac{1}{5(1 - 2y^2)}$

$$\begin{aligned} g(y) &= \frac{1}{5}(1 - 2y^2)^{-1} \\ g'(y) &= \frac{1}{5} \cdot (-1)(1 - 2y^2)^{-2} \cdot (-4y) \\ &= \frac{4y}{5(1 - 2y^2)^2}. \end{aligned}$$

   (c) $P(t) = 5.2e^{-0.4t}$

$$\begin{aligned} P'(t) &= 5.2e^{-0.4t} \cdot (-0.4) \\ &= -2.08e^{-0.4t}. \end{aligned}$$

   (d) $m(z) = \dfrac{-6}{\sqrt[5]{(z^2 + 4)^3}}$

$$\begin{aligned} m(z) &= -6(z^2 + 4)^{-3/5} \\ m'(z) &= -6\left(-\frac{3}{5}\right)(z^2 + 4)^{-8/5} \cdot (2z) \\ &= \frac{36z}{5(z^2 + 4)^{8/5}}. \end{aligned}$$

   (e) $n(t) = \ln\left(e^{7t+1}\right)$

$$\begin{aligned} n(t) &= 7t + 1 \\ n'(t) &= 7. \end{aligned}$$

   (f) $y = 5^{\sqrt{x}}$

$$\begin{aligned} y &= 5^{x^{1/2}} \\ \frac{dy}{dx} &= (\ln 5)\, 5^{\sqrt{x}} \cdot \left(\frac{1}{2}x^{-1/2}\right) \\ &= \frac{(\ln 5)\, 5^{\sqrt{x}}}{2\sqrt{x}}. \end{aligned}$$

   (g) $f(x) = (ax + b)^3(3 - x)$

$$\begin{aligned} f'(x) &= (ax + b)^3 \cdot (-1) + 3(ax + b)^2 \cdot a \cdot (3 - x) \\ &= -(ax + b)^3 + 3a(3 - x)(ax + b)^2 \\ &= (ax + b)^2\left(-(ax + b) + 3a(3 - x)\right) \end{aligned}$$

   Note, we could also simplify further here.

(h) $h(p) = \left(\dfrac{2p-3}{5-p}\right)^7$

$$\begin{aligned} h'(p) &= 7\left(\frac{2p-3}{5-p}\right)^6 \cdot \left(\frac{(5-p)\cdot 2 - (2p-3)\cdot(-1)}{(5-p)^2}\right) \\ &= 7\left(\frac{2p-3}{5-p}\right)^6 \cdot \frac{7}{(5-p)^2} \\ &= 49\left(\frac{2p-3}{5-p}\right)^6 \cdot \frac{1}{(5-p)^2}. \end{aligned}$$

2. Find the first derivative in each. In these problems you need to simplify your answer in a special way. The solutions to these are posted in a video in D2L under Section 3.4.

(a) $g(x) = (2x+1)^4(4x-3)^5$

$$g'(x) = 4(2x+1)^3(4x-3)^4(18x-1).$$

(b) $p(v) = \dfrac{5v}{(v^2+1)^3}$

$$p'(v) = \frac{5-25v^2}{(v^2+1)^4}$$

(c) $r(y) = \dfrac{4y^2}{\sqrt[3]{2-y^2}}$

$$r'(y) = \frac{48y-16y^3}{3(y-y^2)^{4/3}}$$

3. Find each of the indicated derivatives for the function below. The notation in each part tells you which letter represents the independent variable (the remaining letters will be parameters).

$$y = \frac{m}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}.$$

(a) $\dfrac{dy}{dm}$

$$\begin{aligned} y &= \underbrace{\frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}}_{\text{constant}} \cdot m \\ \frac{dy}{dm} &= \frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}. \end{aligned}$$

(b) $\dfrac{dy}{dv}$

$$\begin{aligned} y &= m\left(1-\frac{1}{c^2}\cdot v^2\right)^{-1/2} \\ \frac{dy}{dv} &= m\left(-\frac{1}{2}\right)\left(1-\frac{1}{c^2}\cdot v^2\right)^{-3/2} \cdot \left(-\frac{1}{c^2}\cdot 2v\right). \end{aligned}$$

(c) $\frac{dy}{dc}$

$$\begin{aligned} y &= m(1 - v^2c^{-2})^{-1/2} \\ \frac{dy}{dc} &= m\left(-\frac{1}{2}\right)(1 - v^2c^{-2})^{-3/2}(-v^2 \cdot (-2c^{-3})). \end{aligned}$$