Homework 9

Due Friday 2/3/17

1. Find A^T, A^{-1}, $(A^{-1})^T$, and $(A^T)^{-1}$ for

   (a) $A = \begin{bmatrix} 1 & 0 \\ 9 & 3 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & c \\ c & 0 \end{bmatrix}$

2. Given $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$,

   (a) Compute $(AB)^T$

   (b) Compute $A^T B^T$

3. Suppose a LU factorization of a matrix A is given by

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 9 \\ 0 & 4 \end{bmatrix}.$$

   A LDU factorization can be obtained from this by factoring the pivots from U:

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}.$$

   Here, D is the diagonal matrix with diagonal entries $3, 4$, obtained from the values at the pivot positions in the original U. If we had a LU factorization of a matrix B, as shown below

$$B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 6 \\ 0 & 1 \end{bmatrix},$$

   what would the corresponding D be?

4. Find a LDL^T factorization of $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ by first computing $A = LU$ and then finding $A = LDU$, as above. The name $A = LDL^T$ is due to the fact that the resulting U satisfies $U = L^T$.