Review of Chapter 4 Answer Key

Some Review Problems for Practice

1. Let $g(x) = x^4 + x^3 - 3x^2 + 2$. Be defined on the interval [-2, 1] Using calculus
   - A. Find all the critical points (exact).
   - B. For each critical point indicate if it is a local max, local min or an inflection point.
   - C. Give the critical values for each critical point (round to 3 decimal places.)
   - D. Find all the possible inflection points. Verify that these are inflection points.
   - E. Give the interval(s) that the function, $g(x)$, is increasing and when it is decreasing.
   - F. Give the interval(s) that the function, $g(x)$ is concave up and when it is concave down.
   - G. Find the global max and global min (round to 3 decimal places) [*remember to check the interval given*].

Answer: a&b) local min $x = \frac{-3-\sqrt{105}}{8}$ max $x = 0$ local min $x = \frac{-3+\sqrt{105}}{8}$

c) $g\left(\frac{-3-\sqrt{105}}{8}\right) = -3.248$ $g(0) = 2$ $g\left(\frac{-3+\sqrt{105}}{8}\right) = 0.9549$

d) 0.5 and -1

e) increasing $\left(\frac{-3-\sqrt{105}}{8}, 0\right)$ and $\left(\frac{-3+\sqrt{105}}{8}, 1\right)$

decreasing $\left(-2, \frac{-3-\sqrt{105}}{8}\right)$ and $\left(0, \frac{-3+\sqrt{105}}{8}\right)$

f) concave up (-2, -1) and (0.5, 1) concave down (-1, 0.5)

g) global max value is 2 global min value -3.248

2. Find the exact value of *a* so that $f(t) = te^{at}$ has a critical point at $t = \sqrt{5}$

Answer: $\frac{-1}{\sqrt{5}}$

3. Consider the family of functions $f(x) = e^x + k$. What value of *k* gives the curve that is tangent to the line $y = 4x + 5$? An exact answer will earn full credit.

Answer: $(\ln(4), 4 + k)$ is the point of tangency $k = 4\ln(4) + 1$

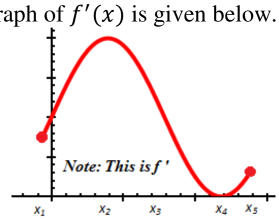
4. Consider the family of functions $f(t) = A\sin(3t) + A\cos(3t) + B\sin(8t) + B\cos(8t)$ where *A* and *B* are parameters. Find exact values of *A* and *B* so that $f(0) = 2$ and $f'(0) = 1$.

Answer: $A = 3$ and $B = -1$

5. The temperature, T, in °C, of a yam put into a 220°C oven is given as a function of time, t, in minutes, by $T(t) = a - ae^{-kt} + b$
   - A. If the yam starts at 25°C, find a and b. (Using algebra and calculus)
   - B. If the temperature of the yam is initially increasing at 3°C per minute, find k.

Answer: $a = 195$ $b = 25$ $k = \frac{3}{195}$

6. The graph of $f'(x)$ is given below.



A. Find the inflection point(s) of $f(x)$.
B. Find the maximum and minimum points of $f(x)$.
C. Give the interval(s) when is $f(x)$ is concave down.

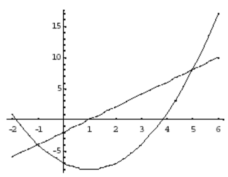
Answer:

A. x_2 and x_4

B. since $f'(x)$ is positive everywhere $f(x)$ in an increasing function so minimum is at x_1 and maximum is at x_5.

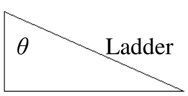
C. (x_2, x_4) $f(x)$ is concave down when $f'(x)$ is decreasing

7. The graph of the function $f(x)$ and its derivative $f'(x)$ are given in the figure. Use this information to find the values of x that maximize and minimize the function $g(x) = f(x)e^{-x}$



Answer: local min at $x = -1$ local max at $x = 5$

8. A 12 foot ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 4 feet per second, how fast is the angle of elevation, θ, changing when the angle is $\frac{\pi}{6}$?



Draw a picture. The angle between the wall and the top of the ladder is θ

The ladder is 12 ft. The length on the ground is x.

The rate of change of the ladder horizontally is $\frac{dx}{dt} = 4\frac{ft}{sec}$.

Find $\frac{d\theta}{dt}|_{\theta=\frac{\pi}{6}}$??

Set up:

$sin\theta = \frac{x}{12}$

When $\theta = \frac{\pi}{6}$ then $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} = \frac{x}{12}$ therefore $x = 6$

Here are two possible methods on finding the rate of change of theta.

Method 1

$\sin\theta = \frac{x}{12}$

$\theta = \arcsin\left(\frac{x}{12}\right)$

Method 2

$\sin\theta = \frac{x}{12}$

$\cos\theta\frac{d\theta}{dt} = \frac{1}{12} * \frac{dx}{dt}$

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$\frac{d\theta}{dt} = \frac{1}{\sqrt{1-\left(\frac{x}{12}\right)^2}} * \frac{1}{12} * \frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{\frac{1}{12}\sqrt{144-x^2}} * \frac{1}{12} * (4)$

(4)

$\frac{d\theta}{dt} = \frac{4}{\sqrt{144-x^2}}$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} * \frac{d\theta}{dt}\Big|_{\theta=\frac{\pi}{6}} = \frac{1}{12} *$

$\frac{d\theta}{dt}\Big|_{\theta=\frac{\pi}{6}} = \frac{2}{3\sqrt{3}}$

Now evaluating when $= \frac{\pi}{6}; x = 6$

$$\frac{d\theta}{dt}\Big|_{\theta=\frac{\pi}{6}} = \frac{4}{\sqrt{144-6^2}}$$

$$\frac{d\theta}{dt}\Big|_{\theta=\frac{\pi}{6}} = \frac{4}{\sqrt{108}} = \frac{4}{6\sqrt{3}}$$

$$\frac{d\theta}{dt}\Big|_{\theta=\frac{\pi}{6}} = \frac{2}{3\sqrt{3}}$$

The angle is changing at a rate of $\frac{2}{3\sqrt{3}}$ $\approx .3849$ radians per second.

9. Find the limits (remember to include your reasons)

	Answer	
A. $\lim_{t\to 0}\frac{\sin t}{7t}$	$\frac{1}{7}$	L'Hopital's does apply
B. $\lim_{t\to\infty}\frac{e^{-5t}}{3t^2+5}$	0	L'Hopital's does not apply type "$\frac{0}{\infty}$"
C. $\lim_{t\to 2}\frac{t^2-4}{3t^2}$	0	L'Hopital's does not apply Use Limit rules.
D. $\lim_{t\to 4}\frac{t^2-16}{t-4}$	8	L'Hopitals does apply.
E. $\lim_{w\to 1}\frac{\ln(w)}{\sqrt{2w^2-2}}$	0	L'Hopital's does apply.

Some more problems from the text for practice:

4.1 #17, 18, 20, 24, 25, 26, 27-30, 33-34, 38-40
4.2 1-25, 22, 27, 40, 41
4.3 Read and set up as many word problems you can.
4.4 20, 30, 32, 33, 34, 37, 51 Show all necessary work
(use calculus for horizontal asymptote)
4.6 32-35
4.7 59, 61,

Review Chapter 4 pages 260-265 (6^{th} edition)
Review of chapter 4: 5, 6, 8, 9, 13, 21, 29, 36, 39, 41, 45, 47, 49, 51, 66, 57, 80, 82-83, 87, 96