Chapter 2 Review Packet (questions 1- 20) KEY

1. In each case determine if the information or statement is correct (C) or incorrect (I). If it is incorrect, include the correction.

I A. $\dfrac{f(a+h)+f(a)}{h}$ represents the slope of the line between points $(a, f(a))$ and $(a+h, f(a+h))$.

I B. Average velocity can be interpreted as the slope of a tangent line.

I C. The difference quotient program finds the exact value of $f'(a)$.

C D. The slope of a function $g(t)$ at $t=3$ can be expressed as $g'(3)$.

C E. Instantaneous velocity can be positive, negative, or zero.

I F. $f'(a)=\lim_{x\to a}\dfrac{f(x+h)-f(x)}{h}$.

C G. The slope of a tangent line is expressed as $\dfrac{\Delta y}{\Delta x}$.

I H.

2.

(i) $T'(1.4)=\dfrac{2.2-1.06}{1.4-1}=2.85$ or $\dfrac{3.2-2.2}{1.8-1.4}=2.5$ or average of the two 2.675

h = .4

(ii) $T'(2.4)=\dfrac{3.1-2.8}{2.6-2.2}=.75$ 2.4 is not on the table so the best average is the smallest h =.2

(iii) $\lim_{h\to 0}\dfrac{T(1.4+h)-T(1.4)}{h}$ = 2.675 or any answer from (i) since it is the same question

(iv) The average rate of change of $T'(x)$ between $x=1.4$ and $x=2.4$.

$$\frac{T'(2.4)-T'(1.4)}{2.4-1.4}=\frac{.75-2.675}{1}=-1.925$$

(v) The rate of change of $T(x)$ at $x=1$. $T'(1)=\dfrac{2.2-1.06}{1.4-1}=2.85$

the smallest h = .4

(vi) Find the equation of the tangent line to $T(x)$ at $x=1$.

Slope = 2.85 the point is (1, 1.06) equation is $y=2.85(x-1)+1.06$

3. $F(x)=10^x$. Estimate $F'(1)$ using a numerical approach (table of values).

Estimate to 3 decimal places, h may have to go very small to achieve this.

See table on the next page

If you only go to -.0001 you can't see the value approaching a number to 3 decimal places, you aren't even sure up to 2 decimal places, so you must go smaller

As h got very small the value seems to be approaching 23.0259 so for 3 decimal places $F'(1)=23.025$ or 23.026 if you rounded to 3 decimal places. Make sure your chart shows the function value approaching this value. You must have at least 4 values that have the same value for 3 decimal places.

h	$\frac{10^{(1+h)}-10}{h}$
-0.1	20.5672
-0.01	22.7628
-0.0001	23.0232
-0.000001	23.0258
-0.0000001	23.0258
-0.00000001	23.0259
0.00000001	23.0259
0.0000001	23.0259
0.000001	23.0259
0.00001	23.0261
0.1	25.8925

4. $G(s)=\frac{1}{s^2}$. Find $G'(2)$ using an algebraic approach.

$$G'(2)=\lim_{h\to0}\frac{G(2+h)-G(2)}{h}$$

$$=\lim_{h\to0}\frac{\frac{1}{(2+h)^2}-\frac{1}{2^2}}{h}=\lim_{h\to0}\frac{4-(2+h)^2}{4h(2+h)^2}=\lim_{h\to0}\frac{4-(4+4h+h^2)}{4h(2+h)^2}=\lim_{h\to0}\frac{-h(4+h)}{4h(2+h)^2}$$

$$=\lim_{h\to0}\frac{-(4+h)}{4(2+h)^2}=\frac{-\lim_{h\to0}(4+h)}{4\left(\lim_{h\to0}(2+h)\right)^2}=\frac{-(4+0)}{4(2+0)^2}=\frac{-1}{4}$$

Just for practice the equation of the tangent line at s = 2 is $y=\frac{-1}{4}x+\frac{3}{4}$

5. The values of the derivative $F'(x)$ are given below:

x	12	12.4	13
$F'(x)$	2	3	3.5

Use this to estimate the values of the function missing in the following table
Must consider what the derivative means change in y for a change in x
So at x = 12 the rate of change is 2, one unit that x increases y increases by 2
We are going only 12.4 not a whole unit; so ratio $\frac{2}{1}=\frac{\Delta y}{.4}$ so $\Delta y=.8$

At 12.4 the rate of change is 3; so $\frac{3}{1}=\frac{\Delta y}{.6}$ so $\Delta y=1.8$

x	12	12.4	13
$F(x)$	8	8.8	10.6

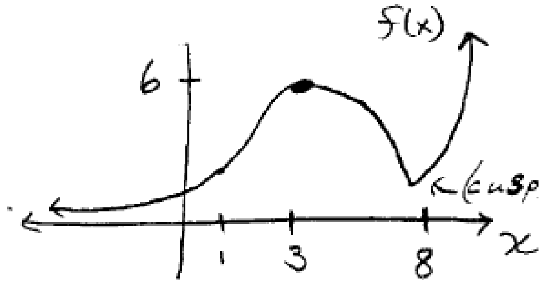
6. Sketch a graph of a function, $f(x)$, with the following properties: **Answers will vary general shape is given.**

$f(3)=6$, $f'(3)=0$, $f'(8)$ is undefined,

$\lim_{x\to-\infty} f(x)=0$, as $x\to\infty$; $f(x)\to\infty$,

$f''(x)>0$ for $x<1$, $x>8$,

$f(x)$ is continuous and defined everywhere.



7. $f'(A)=2.2$ therefore $\dfrac{\Delta y}{\Delta x}=2.2$ $\quad \Delta y = 2.2(.05)=0.11$

(a) $f(A+h)=4+0.11=4.11$ and $f(A-h)=4-0.11=3.89$

(b) smaller since f(x) is concave up.

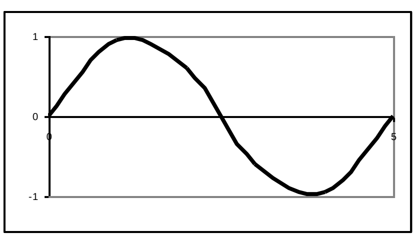
8. Each of the graphs below shows the position of a particle moving in a line as a function of time. During the indicated time interval, which particle has

A) Constant velocity **IV** B) Greatest initial velocity **I** C) Greatest average velocity **III**

D) Zero average velocity **I** E) Zero acceleration **IV** F) Positive acceleration **III**

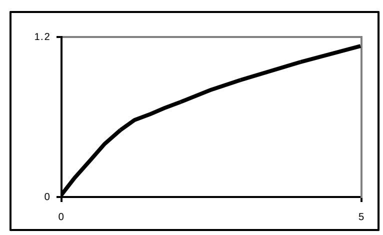
Velocity $\to$ slopes of the distance function
Acceleration $\to$ concavity of distance function

I.



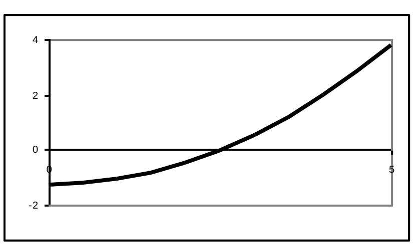
Average rate of change [0,5] =0

II.



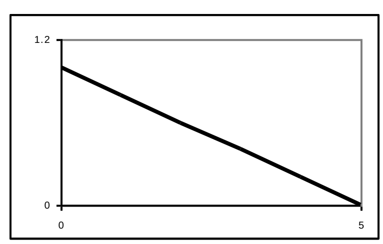
Average rate of change [0,5] =1.2/5 =.24

III.



Average rate of change [0,5] = 5/5 =1

IV.



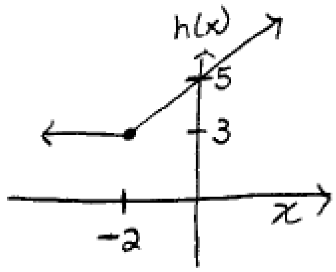
Average rate of change [0,5]= *m* = 1/5 =.2

9. Sketch the graph of $h(x)$ if it has the following characteristics:

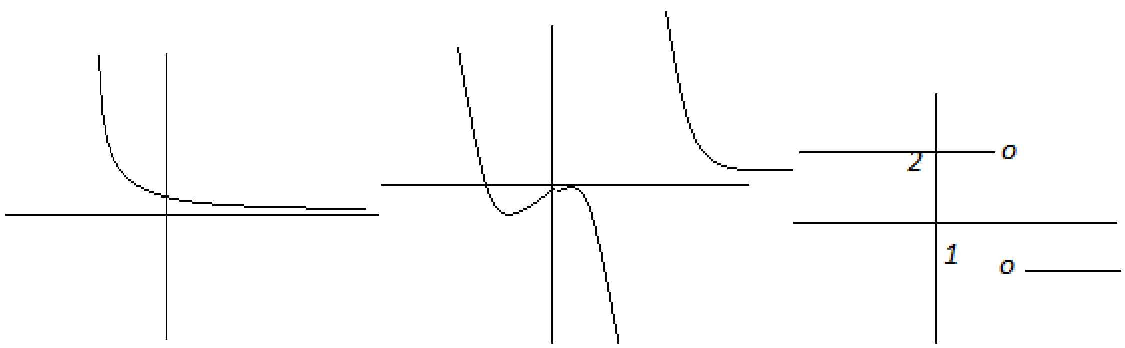
$h'(x)=0$ for *x* in the interval $(-\infty,-2)$ (*horizontal function*)

$h(-2)=3$

$h'(x)=1$ for *x* in the interval $(-2,\infty)$ (*linear function where the slope is 1*)



10.



11. Consider the function $g(x)=\begin{cases} \ln x & x>1 \\ 1.7^x - C & x\le 1 \end{cases}$

A. Determine the value of *C* so that this function is continuous at $x=1$.

$\lim\limits_{x\to 1^+}(\ln(x))=\ln(1)=0$. $\lim\limits_{x\to 1^-}(1.7^x - C)=1.7^1 - C$ To be continuous the function value at *x*=1 must equal 0. $C=1.7$

B. Now determine if this function is differentiable at $x=1$. Prove it.

$x<1$

$$f'(1^-)=\lim_{h\to 0^-}\frac{\left(1.7^{1+h}-1.7\right)-\left(1.7^1-1.7\right)}{h}=\lim_{h\to 0^-}\frac{1.7^{1+h}-1.7}{h}$$

$x>1$

$$f'(1^+)=\lim_{h\to 0^+}\frac{\ln(1+h)-0}{h}=\lim_{h\to 0}\frac{\ln(1+h)}{h}$$

(*next page for the tables*)

h	$\frac{1.7^{(1+h)}-1.7}{h}$
-.001	.90182
-.0001	.90204
-.000001	.902067
-.00000001	.902060

h	$\frac{\ln(1+h)}{h}$
.001	.9995
.00001	.9999995
.0000001	.999999995
.00000001	.999999999995
10^{-11}	1

Since limits are not approaching the same value, the limits of the difference quotient does not exist at *x*=1. That means there are 2 different slopes as *x* approaches 1.
The function is not differentiable at *x* = 1.

12.. Let $p(h)$ be the pressure on a diver (in dynes per square cm) at a depth of *h* meters below the surface of the ocean. Determine what each of the quantities below represent in practical terms. Include units.

A. $p(100)$ the pressure in dynes per square centimeter at 100 meters below the surface

B. $p(h+20)$ the pressure in dynes per square centimeter is the pressure at the divers h meter plus 20 meters below the surface.

C. $p^{-1}(15)$ the depth in meters below the surface where there is 15 dynes per square cm of pressure on the driver.

D. $p'(100)$ the rate (in dynes per square centimeter per meter) at which the pressure is changing when the driver is at 100 meters.

13. Suppose the percent *P* of defective parts produced by a new employee *t* days after the employee starts work can be modeled by $P(t)=\frac{t+1750}{50(t+2)}$.

A. Find $P(30)$ and interpret its meaning in terms of the problem.

$P(30)=\frac{30+1750}{50(30+2)}=1.1125$ A new employee after 30 days will have 1.1125% defective parts.

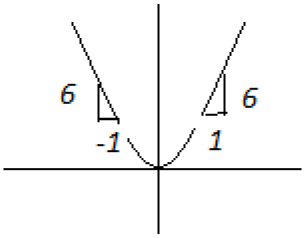
B. Estimate $P'(30)$ and interpret its meaning in terms of the problem.

$$P'(30)=\lim_{h\to 0}\frac{P(30+h)-P(30)}{h}$$ (Use the difference quotient or evaluate function on calculator)

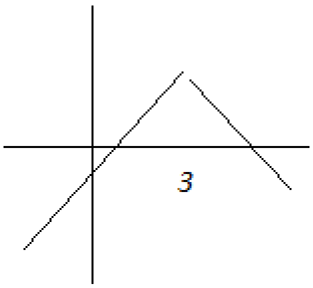
h	$\frac{P(30+h)-P(30)}{h}$
-.000001	-.0341406
-.0000001	-.034141
.00000001	-.034141
.0000001	-.0341406
.00001	-.03414061

At 30 days a new employee's percentage of defective part rate is decreasing .0341 percent per day.

14. (a) $f'(-10) = -6$ (b) $f'(0) = 0$

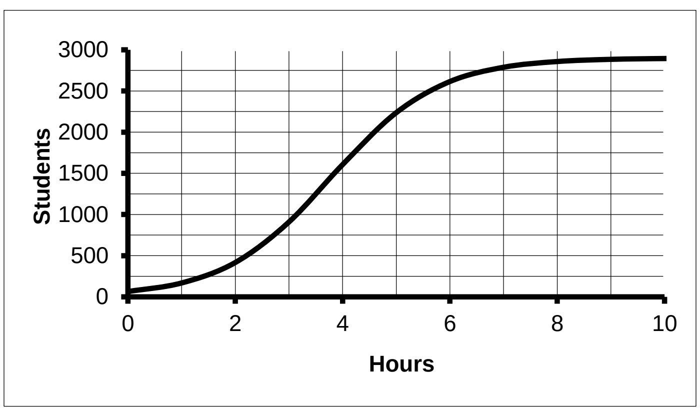


15.



16. (a) Use calculator.
 (b) Use calculator.
 (c) Nothing.
 (d) move values to right by *k.*

17. The registrar has put a counter on the RSVP registration telephone lines to count the total number of students registering during the day. A graph of $N(t)$, the total number of students who have registered during the *t* hours since noon, is given below.



A. Estimate $N'(2)$ and give an interpretation.

Make a tangent line at *t*=2. The points (I found) (3, 750) and (4, 1250) so the slope is 500

At 2 hours after noon or 2pm, the phone registration is increasing 500 calls per hour.

B. Estimate $N^{-1}(2000)$ and give an interpretation.

Find 2000 on the vertical and see where that intersects the function (in 4.5 hours)

At 4:30 pm there have been 2000 students who have registered.

C. Estimate coordinates of the inflection point. Explain the significance of this point in terms of the problem.

3.75 change in concavity.

3.75 hours since noon, at 3:45pm the students registering starts to slow down.

(The rate at which registering students starts to decrease)

18. (a) abortions/year

(b) $N'(4) \approx \dfrac{1,297,606-586,760}{8} = 88,856$. Function is increasing, etc.

(c) $N'(6) \approx \dfrac{1,297,606-988,267}{4} = 77,335$

(d) $N''(6) \approx \dfrac{88,856-77,335}{4} = 2,880$. Concave up – rate of change is increasing.

19. (a) (i) $(-3,-1)\cup(2.5,\infty)$ (ii) $(-\infty,-3)\cup(-1,2.5)$ (iii) -3, -1, 2.5

(b) (i) $(-\infty,-2)\cup(1,4)$ (ii) $(-2,1)\cup(4,\infty)$ (iii) -2, 1, 4

(c) (i) -2 (ii) 0

20.

$$g'(t) = \begin{cases} 1 & t < -1 \\ DNE & t = -1 \\ 3t^2 & -1 < t < 0 \\ 0 & t = 0 \\ 2t & t > 0 \end{cases}$$