Homework 9

§4.1#5, 9, 13, 39, 49, **8**, **22**, **52**

§4.4#23, 31, 33, 49, **28**, **40**, **42**

§4.1 #5. Use derivatives to find critical points and inflection points of the function $f(x) = x^5 - 10x^3 - 8$

§4.1 #9. Find all critical points of the function $f(x) = 3x^4 - 4x^3 + 6$, and then use the first derivative test to determine local maxima and minima.

§4.1 #13. Find the critical points of the function $g(x) = xe^{-3x}$ and classify them as local maxima or local minima or neither.

§4.1 #39. Find the value of a so that the function $f(x) = xe^{ax}$ has a critical point at $x = 3$

§4.1 #49. The differentiable function f has $x = 1$ as its only zero and $x = 2$ as its only critical point. Find all a) zeros and b) critical points of the function $y = f(x^2 - 3)$

§4.1 #**8**. Use derivatives to find the critical points and inflection points of the function $f(x) = 4xe^{3x}$.

§4.1 #**22**. a) If a is a nonzero constant, find all critical points of $f(x) = \frac{a}{x^2} + x$

b) Use the second derivative test to show that if a is positive then the graph has a local minimum, and if a is negative then the graph has a local maximum.

§4.1 #**52**. Consider a function f, the graph of which lies entirely above the x-axis and the derivative of which is always negative, $f'(x) < 0$. If $y = f(x^2)$

a) Give the critical point(s) of y, or explain how you know there are none, and

b) Say where the function y increases and where it decreases.

§4.4 #23. a) Graph $f(x) = x + a\sin(x)$ for $a = 0.5$ and $a = 3$.

b) For what values of a is $f(x)$ increasing for all x?

§4.4 #31. Let $g(x) = x - ke^x$, where k is a constant. For what values of k does the function g have a critical point?

§4.4 #33. Find a function of the form $y = be^{-(x-a)^2/2}$ with its maximum at the point $(0, 3)$.

§4.4 #49. a) Find all critical points of $f(x) = x^4 + ax^2 + b$

b) Under what conditions on a and b does this function have exactly one critical point? What is the one critical point, and is it a local maximum, a local minimum, or neither?

c) Under what conditions on a and b does this function have exactly 3 critical points? What are they? Which are local maxima and which are local minima?

d) Is it ever possible for this function to have two critical points? No critical points? More than three critical points? Explain.

§4.4 #**28**. If $a > 0$ and $b > 0$, show that $f(x) = a(1 - e^{-bx})$ is everywhere increasing and everywhere concave down.

§4.4 #**40**. Find a function of the form $y = a\cos(bt^2)$ whose first critical point for positive t occurs at $t = 1$ and whose derivative is -2 when $t = 1/\sqrt{2}$.

§4.4 #**42**. Find a function of the form $y = bxe^{-ax}$ with a local maximum at (3,6).