Homework 2

§2.3#1, 3, 17, 23, 47, 51, **8**, **24**

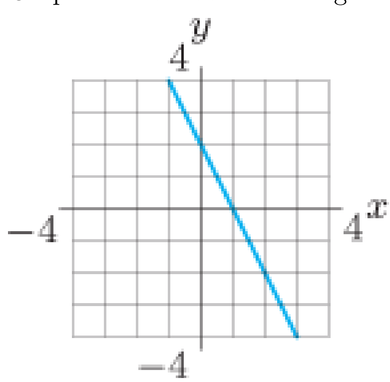
§2.6#8*, 10, 13, **16***, **Extra**

§2.3 #1. a) Estimate $f'(2)$ using the values of f in the table.

b) For what values of x does $f'(x)$ appear to positive? Negative?

x	0	2	4	6	8	10	12
$f(x)$	10	18	24	21	20	18	15

§2.3 #3. Graph the derivative of the given function:



§2.3 #17. Sketch a graph of $f(x) = \cos(x)$ and use this graph to sketch $f'(x)$.

§2.3 #23. In each case, graph a smooth curve whose slope meets the condition:

a) Everywhere positive and increasing gradually

b) Everywhere positive and decreasing gradually

c) Everywhere negative and increasing gradually (becoming less negative)

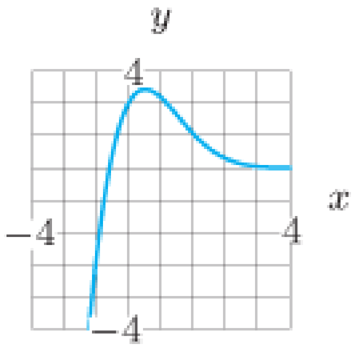
d) Everywhere negative and decreasing gradually (becoming more negative)

§2.3 #47. Draw the graph of a continuous function $y = f(x)$ that satisfies the following three conditions:

- $f'(x) > 0$ for $x < -2$
- $f'(x) < 0$ for $-2 < x < 2$
- $f'(x) = 0$ for $x > 2$

§2.3 #51. Using a graph, explain why if $g(x)$ is an odd function, then $g'(x)$ is even.

§2.3 #**8**. Sketch a graph of the derivative $f'(x)$ given the graph of the function $f(x)$ below.



§2.3 #**24**. Sketch a function with the given properties:

- $f'(x) > 0$ for $x < -1$
- $f'(x) < 0$ for $x > -1$
- $f'(x) = 0$ at $x = -1$

§2.6 #8*. In each of the following cases, sketch the graph of a continuous function $f(x)$ with the given properties:

a) $f'(x) > 0$ for $x < 2$ and for $x > 2$ and $f'(2)$ is undefined

b) $f'(x) > 0$ for $x < 2$ and $f'(x) < 0$ for $x > 2$ and $f'(2)$ is undefined

§2.6 #10. The acceleration due to gravity, g, varies with height above the surface of the earth. If you go down below the surface of the earth, g varies in a different way. It can be shown that g is given by

$$g = \begin{cases} \frac{GMr}{R^3} & r < R \\ \frac{GM}{r^2} & r \geq R \end{cases}$$

where R is the radius of the earth, M is the mass of the earth, G is the gravitational constant, and r is the distance to the center of the earth.

a) Sketch a graph of g against r

b) Is g a continuous function of r? Explain your answer.

c) Is g a differentiable function of r? Explain your answer.

§2.6 #13. A cable is made of an insulating material in the shape of a long, thin cylinder of radius r_0. It has electric charge distributed evenly throughout it. The electric field, E, at a distance r from the center of the cable is given by

$$E = \begin{cases} kr & r \le r_0 \\ k\frac{r_0^2}{r} & r > r_0 \end{cases}$$

a) Is E continuous at $r = r_0$?

b) Is E differentiable at $r = r_0$?

c) Sketch a graph of E as a function of r.

§2.6 #**16***. Sometimes, odd behavior can be hidden beneath the surface of a rather normal looking function. Consider the following function:

$$f(x) = \begin{cases} 0 & if\ x < 0 \\ x^2 & if\ x \geq 0 \end{cases}$$

a) Sketch a graph of this function. Does it have any vertical segments of corners? Is it differentiable everywhere? If so, sketch the derivative f' of this function

b) Is the derivative function, $f'(x)$ differentiable everywhere? If not, at what point(s) is it not differentiable? Draw the second derivative wherever it exists. (The second derivative is just the derivative of the function $f'(x)$. It is usually denoted $f''(x)$.) Is the second derivative function, $f''(x)$, differentiable? Continuous?

Extra. The function $f(x) = x^{2/3}$ has a cusp at $x = 0$.

a) Sketch a graph of $f(x)$.

b) Using the limit definition of the derivative, show that $f(x)$ is not differentiable at $x = 0$.