Section 4.2

Objectives

- Set up a quadratic function to model certain "real life" situations.
- Use quadratic models and interpret results in "real life" terms.
- Solve application problems involving quadratic functions generated from projectile motion models, area models, and demand/ cost/ revenue/profit models.

Preliminaries

Summarize key formulas from this section:

Area formulas for the following shapes

Rectangle $A = BH$ Area = Base · Height

Circle

Surface area formulas

Rectangular box

Cylinder

Business model definitions

The demand function models price as a function of quantity.

The revenue from selling x items at a price p is given by the formula pX.

Warm-up

1. A rectangle has a perimeter of 10 inches. Express the area of the rectangle as a function of its width.

$A = LW$

$P = 10 = 2L + 2W$

Solve for L in the second equation.

$\frac{2L}{2} = \frac{10-2W}{2}$

$L = \frac{10-2W}{2}$

$L = 5 - W$

Plug $5-W$ in for L in the first equation

$A = (5-W)W$

$A = 5W - W^2$

$A = -W^2 + 5W.$

2. Suppose the fixed cost of producing a line of sunglasses is $25,000 and each pair of sunglasses costs $3 to make. Find a formula for the total cost of producing s pairs of sunglasses.

Total Cost = Fixed cost + Production cost

$C(x) = 25{,}000 + 3x$

Class Notes and Examples

4.2.1 A stone is thrown upward; its height in meters t seconds after release is given by

$$h(t) = -4.9t^2 + 49t + 277.4.$$

(A) How high was the stone when it was released?

The stone was released "0 seconds after release", $t=0$.

The height of the stone when $t=0$ is $h(0) = -4.9(0)^2 + 49(0) + 277.4$
$= 277.4$.

The stone was 277.4 meters high when it was released.

(B) How long will it take the stone to hit the ground? (Round to the nearest 0.01 second)

When the stone hits the ground, its height is zero.
So set $h(t)$ equal to zero and solve for t.

$0 = -4.9t^2 + 49t + 277.4$

Use the quadratic formula to solve for t.

$t \approx 14.0339$ or $t \approx \cancel{-4.0339}$ (t must be greater than zero.)

It will take 14.03 seconds for the stone to hit the ground, rounded to the nearest 0.01 second.

(C) When will the stone be at its maximum height?

The maximum height of the stone occurs at the vertex.
The x-coordinate of the vertex is the time when the stone will be at its maximum height.
The x-coordinate of the vertex is $-\frac{b}{2a} = \frac{-49}{2(-4.9)} = \frac{10}{2} = 5$

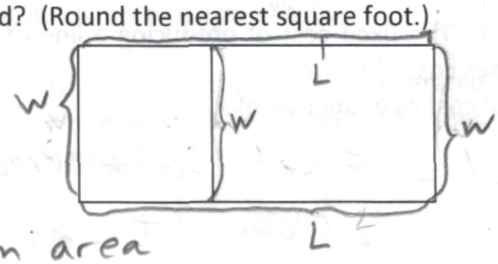
The stone will be at its maximum height 5 seconds after being released.

(D) What is the maximum height? Verify by graphing.

The maximum height is the y-coordinate of the vertex.
The y-coordinate of the vertex is $h(5) = -4.9(5)^2 + 49(5) + 277.4$
$= 399.9$

The maximum height is 399.9 meters.

4.2.2 A rancher has 10,000 feet of fencing and wants to enclose a rectangular field with an internal fence parallel to one side, as shown below. What is the maximum total area that can be enclosed? (Round the nearest square foot.)



Want: Maximum area

Variables: Let A square feet be the area enclosed.
Let L feet be the length
Let W feet be the width

Equations: $A = LW$ ← Geometry fact
$10{,}000 = 2L + 3W$ ← Amount of fencing.

Our goal is to get an equation of the for $A = aW^2 + bW + c$ so that we can find its vertex, and thus, the maximum area.

First solve for L in the equation $10{,}000 = 2L + 3W$

$10{,}000 = 2L + 3W \longrightarrow 2L = 10{,}000 - 3W \longrightarrow L = \frac{10{,}000 - 3W}{2} = -\frac{3}{2}W + 5{,}000$

Second plug $-\frac{3}{2}W + 5{,}000$ in for L in the equation $A = LW$

$$A = \left(-\frac{3}{2}W + 5{,}000\right)W = -\frac{3}{2}W^2 + 5{,}000W$$

The x-coordinate of the vertex is $-\frac{b}{2a} = \frac{-5000}{2(-\frac{3}{2})} = \frac{-5000}{-3} = \frac{5000}{3}$

The y-coordinate of the vertex is the maximum area that can be enclosed. It is $-\frac{3}{2}\left(\frac{5000}{3}\right)^2 + 5000\left(\frac{5000}{3}\right) \approx 4{,}166{,}666.667$

So the maximum total area that can be enclosed is 4,166,667 square feet (rounded to the nearest square foot)

4.2.3 Suppose a sunglass manufacturer determines the demand function for a certain line of sunglasses is given by $p = 50 - \frac{1}{4{,}000}x$, where p is the price per pair and x is the number of pairs sold. The fixed cost of producing a line of sunglasses is \$25,000 and each pair of sunglasses costs \$3 to make.

(A) Express the total cost as a function of x.

Total cost = Fixed Cost + Production Cost

$C(x) = 25{,}000 + 3x$

(B) Express the revenue as a function of x.

Revenue = px

$R(x) = \left(50 - \frac{1}{4000}x\right)x$

$R(x) = -\frac{1}{4000}x^2 + 50x$

(C) Express the profit as a function of x.

Profit = Revenue − Cost

$P(x) = \left(-\frac{1}{4000}x^2 + 50x\right) - (25{,}000 + 3x)$

$= -\frac{1}{4000}x^2 + 47x - 25{,}000$

(D) How many sunglasses should be produced to maximize profit?

The number of sunglasses that should be produced and sold to maximize profit is the x-coordinate of the parabola $P(x) = -\frac{1}{4000}x^2 + 47x - 25{,}000$.

The x-coordinate of the vertex is

$$-\frac{b}{2a} = \frac{-47}{2\left(-\frac{1}{4000}\right)} = \frac{-47}{-\frac{1}{2000}}$$

$$= -47 \cdot \frac{-2000}{1}$$

$$= 47 \cdot 2000$$

$$= 94{,}000$$

94,000 pairs of sunglasses should be produced.

4.2.4 A concert venue holds a maximum of 1000 people. With ticket prices at \$30, the average attendance is 650 people. It is predicted that for each dollar the ticket price is lowered, approximately 25 more people attend.

(A) Create a function to represent the revenue generated from ticket sales. (You will need to decide what your variables should represent. Creating a table of values may help you get started.)

p = Ticket price	# People attending = x
\$30	650
\$29	675
\$28	700

These points make a line.

$$x - 650 = -25(p - 30)$$
$$x = -25p + 1400$$

$$\text{Revenue} = px = p(-25p + 1400)$$
$$R(x) = -25p^2 + 1400p$$

(B) What is the maximum possible revenue from this concert? How should the tickets be priced and how many people will attend at that price?

The y-coordinate of the vertex of the parabola $R(x) = -25p^2 + 1400p$ is the maximum possible revenue.

The x-coordinate is $-\frac{b}{2a} = \frac{-1400}{2(-25)} = \frac{-1400}{-50} = 28$

The y-coordinate is $R(28) = -25(28)^2 + 1400(28) = 19{,}600$

The maximum possible revenue is \$19,600.

Tickets should be priced at \$28. (The x-coordinate of the vertex)

By the table in part (A), 700 people will attend if the ticket price is \$28.