CHAPTER 4

Section 4.1

Objectives

- Given a function represented algebraically, determine whether the function is quadratic.
- Convert a quadratic function from general form to standard form.
- Identify and understand the components of a quadratic function in standard form, and identify the impact of these components on the graph of the function.
- Complete the square for a quadratic function in general form.
- Identify the vertex or maximum/minimum of a quadratic function using either the standard form of the function or the vertex formula.
- Given the graph of a quadratic function, find the equation of the function.
- Find the equation of a quadratic function, given the vertex and one other point.
- Find a possible equation of a quadratic function, given two zeros.

Preliminaries

The solutions to $ax^2 + bx + c = 0$, with $a \neq 0$, are given by $x =$ $\underline{\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}}$

Warm-up

1. Solve the equation by completing the square: $x^2 - 2x = 7$

2. Consider $y = -2(x-1)^2 + 3$.
   (A) Identify an appropriate base graph and describe the transformations.

   (B) Sketch an accurate graph of $y = -2(x-1)^2 + 3$.

Class Notes and Examples

A quadratic function is a function of the form $y = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0. There are two other useful ways in which quadratics can be represented: standard form and factored form.

General form: $y = ax^2 + bx + c$

The parabola opens upward if $a > 0$; downward if $a < 0$.

The y intercept is $(0, c)$.

the x-coordinate of the vertex is $x = -\frac{b}{2a}$

the y-coordinate of the vertex is $y = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c$

Just plug in $-\frac{b}{2a}$ for the x-value.

Factored form: $y = a(x - r_1)(x - r_2)$

The parabola opens upward if $a > 0$; downward if $a < 0$.

The x-intercepts are $(0, r_1)$ and $(0, r_2)$.

The vertex x-coordinate of the vertex is $\frac{r_1 + r_2}{2}$.

Standard form: $y = a(x - h)^2 + k$

The parabola opens upward if $a > 0$; downward if $a < 0$.

The vertex is at (h, k)

If the parabola opens upward, the function has a minimum value at $x = h$.

If the parabola opens downward, the function has a maximum value at $x = h$.

The axis of symmetry contains the vertex, with equation $x = h$.

4.1.1 Consider the quadratic function $f(x) = 4x^2 - 7x - 2$

(A) Write $f(x)$ in factored form.

Using part (B), we know that $(2,0)$ and $(-1/4, 0)$ are x-intercepts. So $r_1 = 2$ and $r_2 = -1/4$. Thus $f(x) = a(x-2)(x+1/4)$ in factored form. we only need to find "a". After multiplying out $a(x-2)(x+1/4)$ we find that $f(x) = ax^2 + \dots$, and we know $f(x) = 4x^2 + \dots$ Thus $a=4$.

So $f(x) = 4(x-2)(x+1/4)$ in factored form

(B) Determine the intercepts.

To find the y-intercept, set $x=0$. $f(0) = 4(0)^2 - 7(0) - 2 = -2$

So the y-intercept is $(0,-2)$.

To find the x-intercepts set $f(x) = 0$.

$0 = 4x^2 - 7x - 2$. $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-2)}}{2(4)} = \dfrac{7 \pm \sqrt{81}}{8} = \dfrac{7 \pm 9}{8} = 2$ or $-\frac{1}{4}$

The x-intercepts are $(2,0)$ and $(-\frac{1}{4}, 0)$.

(C) Determine the vertex.

The x-coordinate of the vertex is $\dfrac{-b}{2a} = \dfrac{-(-7)}{2 \cdot (4)} = \dfrac{7}{8}$

The y-coordinate of the vertex is $f(\frac{7}{8}) = 4(\frac{7}{8})^2 - 7(\frac{7}{8}) - 2 = -5\frac{1}{16}$

The vertex is at $(\frac{7}{8}, -5\frac{1}{16})$

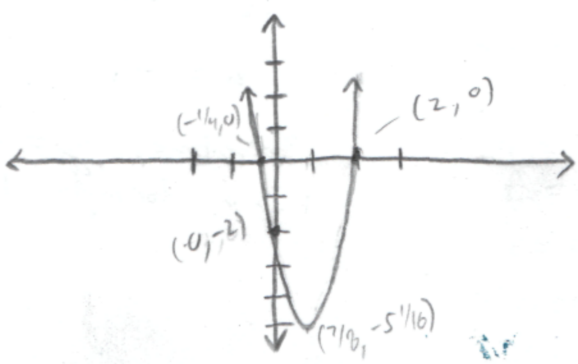
(D) Does the parabola open up or down?

$a = 4 > 0$. So the parabola opens up.

(E) Is the vertex a maximum or minimum?

The vertex is a minimum.

(F) Sketch a graph of the parabola.



4.1.2 Consider the quadratic function $f(x) = 3x^2 - 24x + 2$. Use two different methods to determine the vertex of the function.

The x-coordinate of the vertex is $\frac{-b}{2a} = \frac{-(-24)}{2(3)} = \frac{24}{6} = 4$.

The y-coordinate of the vertex is $f(4) = 3(4)^2 - 24(4) + 2$

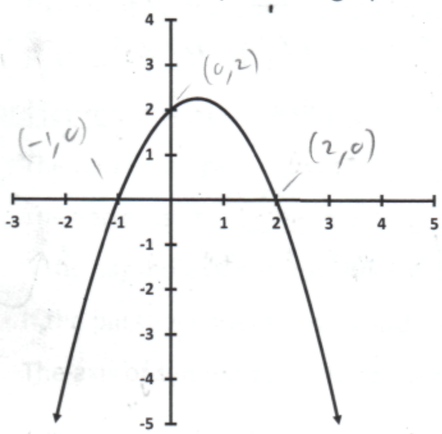
$= 3 \cdot 16 - 24 \cdot 4 + 2$

$= 48 - 96 + 2$

$= -46$.

The vertex of the parabola is $(4, -46)$

4.1.3 Find a formula the parabola graphed below.



We can find the x-intercepts of the parabola. So, it would be easiest to use factored form

$r_1 = -1$ and $r_2 = 2$

Thus, the equation of the parabola is $y = a(x+1)(x-2)$.

To find "a", plug the point $(0,2)$ into the equation and solve for "a".

$2 = a(0+1)(0-2)$

$2 = a(1)(-2)$

$\frac{2}{-2} = \frac{-2a}{-2}$

$a = -1$

Therefore, $y = -(x+1)(x-2)$ is a formula for the parabola.

4.1.4 Find a formula for a parabola that goes through the points $(-5, 0)$, $(3, 0)$ and $(1, 16)$.

The x-intercepts of the parabola are given. So, it would be easiest to use factored form. $r_1 = -5$ and $r_2 = 3$.

Thus, the equation of the parabola is $y = a(x+5)(x-3)$.

To find "a", plug the point $(1,16)$ into the equation and solve for "a".

$16 = a(1+5)(1-3)$

$16 = a(6)(-2)$

$\frac{16}{-12} = \frac{-12a}{-12}$

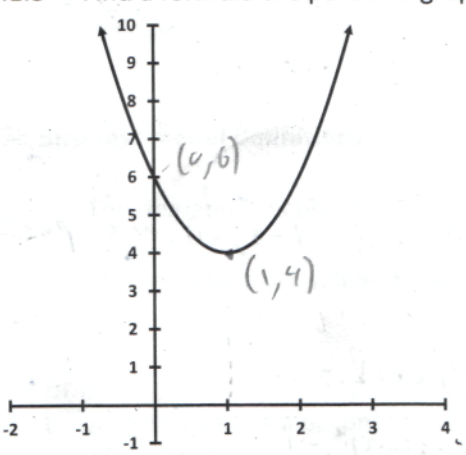
$a = \frac{16}{-12}$

$a = -\frac{4}{3}$

Therefore, the equation of the parabola is

$y = -\frac{4}{3}(x+5)(x-3)$.

4.1.5 Find a formula the parabola graphed below.



We can find the vertex of the parabola. So it would be easiest to use standard form.

$(h,k) = (1,4)$ $h = 1$ $k = 4$.

Thus, the equation of the parabola is $y = a(x-1)^2 + 4$

To find "a", plug the point $(0,6)$ into the equation and solve for "a".

$6 = a(0-1)^2 + 4$

$6 = a(-1)^2 + 4$

$6 = a + 4$

$-4 \quad -4$

$2 = a$

$a = 2$

Therefore, $y = 2(x-1)^2 + 4$ is a formula for the parabola.

4.1.6 Find a formula for a parabola whose vertex is at $(-2, -1)$ and goes through the point $(0, 11)$. Check your solution.

The vertex of the parabola is given. So it would be easiest to use standard form. $(h, k) = (-2, -1)$ $h = -2$ $k = -1$.

Thus the equation of the parabola is $y = a(x+2)^2 - 1$

To find "a", plug the point $(0, 11)$ into the equation and solve for "a".

$$11 = a(0+2)^2 - 1$$
$$11 = a(2)^2 - 1$$
$$11 = 4a - 1$$
$$+1 \qquad +1$$
$$\frac{12}{4} = \frac{4a}{4}$$
$$a = 3$$

Therefore, $y = 3(x+2)^2 - 1$ is an equation for the parabola.

Check:
Let's verify that both $(0, 11)$ and $(-2, -1)$ are points on the parabola.

$y = 3(x+2)^2 - 1$	$y = 3(x+2)^2 - 1$
$11 \stackrel{?}{=} 3(0+2)^2 - 1$	$-1 \stackrel{?}{=} 3((-2)+2)^2 - 1$
$11 \stackrel{?}{=} 3 \cdot 2^2 - 1$	$-1 \stackrel{?}{=} 3(0)^2 - 1$
$11 \stackrel{?}{=} 3 \cdot 4 - 1$	$-1 \stackrel{?}{=} 3 \cdot 0 - 1$
$11 \stackrel{?}{=} 12 - 1$	$-1 \stackrel{?}{=} 0 - 1$
$11 \stackrel{\checkmark}{=} 11$	$-1 \stackrel{\checkmark}{=} -1$

Both points are on the parabola.