Objectives

- Find the sum, difference, product, or quotient of functions represented graphically, algebraically, or in table form.
- Find the domain of the sum, difference, product, or quotient of functions.
- Compose two functions represented graphically, algebraically, or in table form.
- Given a compound function, identify two functions whose composition yields the given function.
- Find the domain of a composition of two functions.

Preliminaries

Complete the following definitions

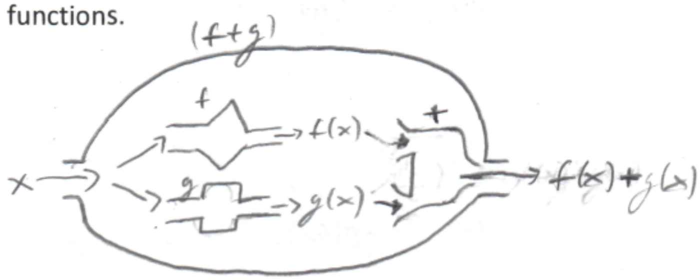
$(f+g)(x) =$ $f(x)+g(x)$

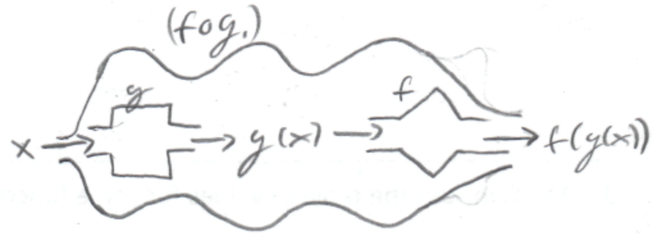
$(f-g)(x) =$ $f(x)-g(x)$

$(fg)(x) =$ $f(x)\cdot g(x)$

$(f/g)(x) =$ $f(x)/g(x)$

$(f \circ g)(x) =$ $f(g(x))$





Warm-up

1. Find the interval(s) for all x values that satisfy both of the following inequalities: $x \neq 2$ and $x \leq 8$

2. Find the domain of each of the following functions.

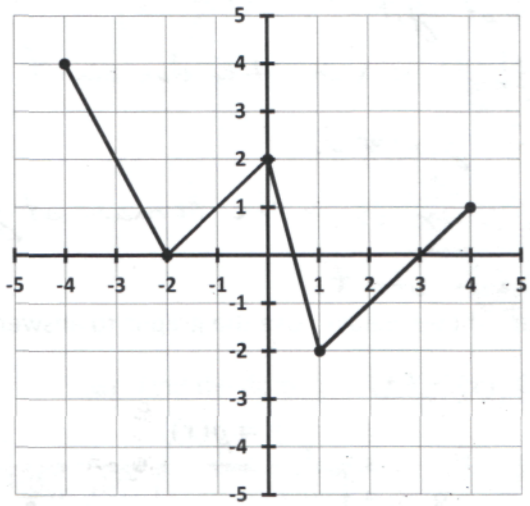
   (A) $f(x) = \sqrt{3x-2}$

   (B) $g(x) = \frac{x-1}{x^2-x-6}$

Class Notes and Examples

3.5.1 Use the functions $h(x)$, $g(x)$, and $f(x)$ given below to determine the following values if they are defined:

$h(x)$



$f(x) = 3x - 4$

x	-2	0	2	3	4
$g(x)$	4	-3	3	6	1

(A) $\left(\frac{h}{f}\right)(3) = \frac{h(3)}{f(3)}$

$= \frac{0}{3\cdot(3)-4} = \frac{0}{5} = 0$

(B) $(h+g)(0) = h(0)+g(0)$

$= 2+(-3)$

$= 2-3 = -1$

(C) $\left(\frac{g}{h}\right)(3) = \frac{g(3)}{h(3)}$ Nonsense

$= \frac{6}{0}$ You can't divide by zero.

3 is not in the domain of ~~g~~h.

(D) $(fg)(0) = f(0)\cdot g(0)$

$= (3\cdot(0)-4)\cdot(-3)$

$= (-4)\cdot(-3) = 12$

(E) $(g-h)(2) = g(2)-h(2)$

$= 3-(-1)$

$= 3+1 = 4$

(F) $(h \circ g)(1) = h(g(1))$

Nonsense.

1 is not in the domain of g.

(G) $(f \circ g)(4) = f(g(4))$

$= f(1)$

$= 3\cdot(1)-4$

$= -1$

(H) $(h \circ h)(0) = h(h(0))$

$= h(2)$

$= -1$

❓ In general, how do you find the domain of $f+g, f-g, fg, \frac{f}{g}$, and $f \circ g$ given the domain of f and the domain of g?

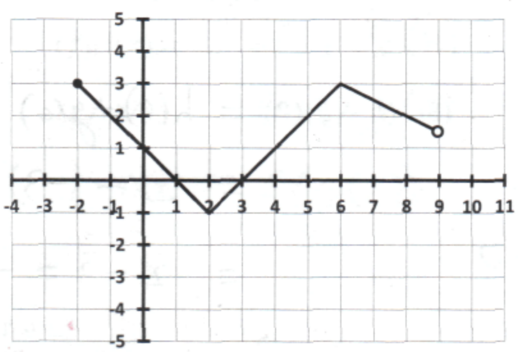
The domain of $f+g$, $f-g$, and fg is all x such that x is in the domain of f and in the domain of g.

The domain of f/g is all x such that x is in the domain f and in the domain of g and such that $g(x) \neq 0$.

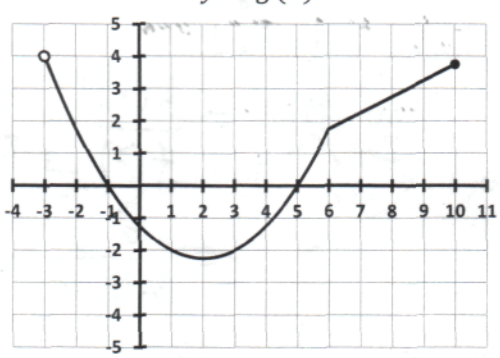
The domain of $f \circ g$ is all x such that x is in the domain of g and such that $g(x)$ is in the domain of f.

3.5.2 The graphs of the functions $f(x)$ and $g(x)$ are shown below. Use the graphs to answer the questions below.

$y = f(x)$



$y = g(x)$



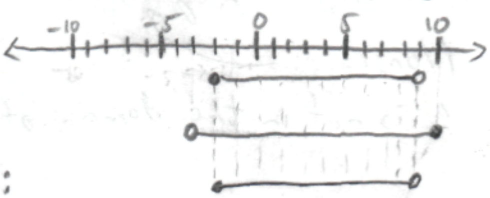
(A) Find the domain of f.

The domain of f is $[-2, 9)$

(B) Find the domain of g.

The domain of g is $(-3, 10]$

(C) Find the domain of $f+g$.



Domain of f:

Domain of g:

All x in both:

The domain of $f+g$ is $[-2, 9)$.

(D) Find the domain of fg.

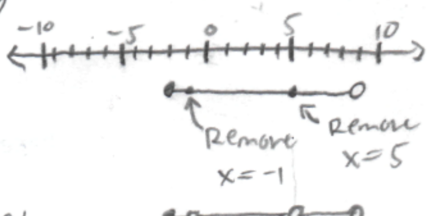
Same as for $f+g$.

$[-2, 9)$.

(E) Find the domain of $\frac{f}{g}$.

$g(x) = 0$ when $x = -1$ or 5

All x in both:

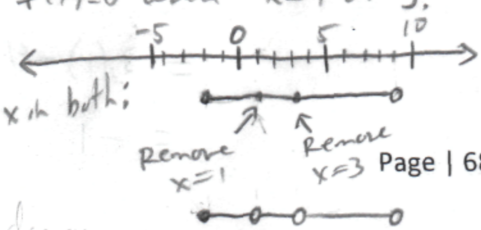


The domain of f/g is $[-2,-1) \cup (-1,5) \cup (5,9)$

(F) Find the domain of $\frac{g}{f}$.

$f(x) = 0$ when $x = 1$ or 3.

All x in both:



The domain of g/f is $[-2,1) \cup (1,3) \cup (3,9)$

3.5.3 Consider the functions $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{x-2}{x+5}$.

(A) Find $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x))$$
$$= f\left(\frac{x-2}{x+5}\right)$$
$$= \frac{\left(\frac{x-2}{x+5}\right) - 1}{\left(\frac{x-2}{x+5}\right)}$$
$$= \frac{\frac{x-2}{x+5} - \frac{x+5}{x+5}}{\frac{x-2}{x+5}}$$
$$= \frac{\left(\frac{(x-2)-(x+5)}{x+5}\right)}{\left(\frac{x-2}{x+5}\right)}$$
$$= \frac{\left(\frac{-7}{x+5}\right)}{\left(\frac{x-2}{x+5}\right)}$$
$$= \left(\frac{x+5}{x-2}\right)\left(\frac{-7}{x+5}\right)$$
$$= \frac{-7}{x-2}$$

(B) Find the domain of $(f \circ g)(x)$. Check your solution.

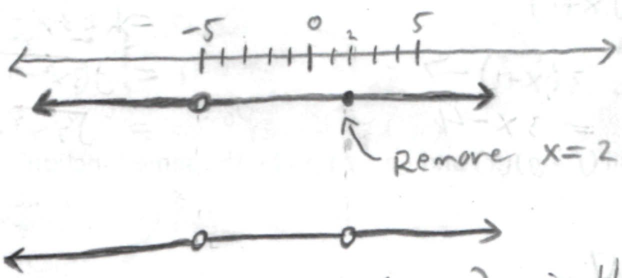
First, find the domain of g. The domain of g is all real numbers except $x=-5$. In interval notation this is $(-\infty,-5)\cup(-5,\infty)$.

Second, find the domain of f. The domain of f is all real numbers except $x=0$.

Third, find all x such that $g(x)$ is not in the domain of f. In this case, we are to find all x such that $g(x)=0$.

Solve for x in the equation $0 = \frac{x-2}{x+5}$. $x=2$.

Fourth, remove all points you found in the third step from the domain of g.



Domain of g:

Remove $x=2$.

Domain of $f \circ g$:

$(-\infty,-5)\cup(-5,2)\cup(2,\infty)$ is the domain of $f \circ g$.

3.5.4 Use the functions $f(x) = 3x^2 - 7$ and $h(x) = \sqrt{x+1}$ to determine the following. Indicate the domain of each function.

(A) $(f - h)(x) = f(x) - h(x)$

$= (3x^2 - 7) - \sqrt{x+1}$

$= 3x^2 - 7 - \sqrt{x+1}$

Domain f: $(-\infty, \infty)$

Domain h: $[-1, \infty)$

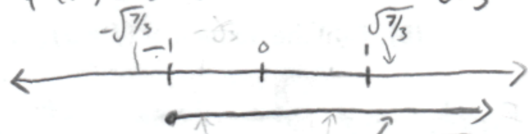
Domain $f-h$: $[-1, \infty)$

(B) $\left(\frac{h}{f}\right)(x) = \dfrac{h(x)}{f(x)}$

$= \dfrac{\sqrt{x+1}}{3x^2 - 7}$

All x in both domain of f and h: $[-1, \infty)$

$f(x) = 0$ when $x = \pm\sqrt{\frac{7}{3}}$



Remove $x = \sqrt{\frac{7}{3}}$

Domain $\frac{h}{f}$: $[-1, \sqrt{\frac{7}{3}}) \cup (\sqrt{\frac{7}{3}}, \infty)$

(C) $(f \circ h)(x)$

Domain h: $[-1, \infty)$

Domain f: $(-\infty, \infty)$.

There are no x such that $h(x)$ is not in the domain of f.

So we remove nothing from the domain of h.

Domain $f \circ h$: $[-1, \infty)$

$(f \circ h)(x) = f(h(x)) = f(\sqrt{x+1})$

$= 3(\sqrt{x+1})^2 - 7 = 3(x+1) - 7$

$= 3x - 4$

(D) $(h \circ f)(x)$

Domain f: $(-\infty, \infty)$

Domain h: $[-1, \infty)$.

Find all x such that $f(x)$ is in the domain of $h(x)$.

All x such that $3x^2 - 7 \geq -1$

$x^2 \geq 2$

$x \leq -\sqrt{2}$ or $x \geq \sqrt{2}$

Domain of $(h \circ f)(x)$: $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

$(h \circ f)(x) = h(f(x))$

$= h(3x^2 - 7)$

$= \sqrt{(3x^2 - 7) + 1}$

$= \sqrt{3x^2 - 6}$

In general, will $(f \circ g)(x)$ and $(g \circ f)(x)$ be the same function?

No.

3.5.5 For each part below, suppose $h(x) = (f \circ g)(x)$.

(A) Find $f(x)$ if $h(x) = (x^2 - 1)^3 + 4$ and $g(x) = x^2$.

$h(x) = (f \circ g)(x)$

$h(x) = f(g(x))$

$(x^2-1)^3 = f(x^2)$

That is, replacing every x in $f(x)$ by x^2 should get us $(x^2-1)^3$. So $f(x) = (x-1)^3$.

(B) Find $g(x)$ if $h(x) = \frac{4}{x^2+x}$ and $f(x) = \frac{4}{x}$.

$h(x) = (f \circ g)(x)$

$h(x) = f(g(x))$

$\frac{4}{x^2+x} = \frac{4}{g(x)}$

Solve So $g(x) = x^2 + x$.

(C) If $h(x) = \sqrt{2x+1}$, what could $f(x)$ and $g(x)$ be? Check your solution.

$h(x) = (f \circ g)(x)$

$h(x) = f(g(x))$

$\sqrt{2x+1} = f(g(x))$

outside. inside

Inside stuff is $g(x)$.

outside stuff is $f(x)$.

$f(x) = \sqrt{x}$ $g(x) = 2x+1$

$(f \circ g)(x) = f(g(x)) = f(2x+1) = \sqrt{2x+1}$

Alternative solutions:

$f(x) = \sqrt{x+1}$ $g(x) = 2x$

$f(x) = \sqrt{2x+1}$ $g(x) = x$

$f(x) = x$ $g(x) = \sqrt{2x+1}$

$f(x) = \sqrt{2x}$ $g(x) = x + \frac{1}{2}$