

## Section 3.4

---

### Objectives

- Given a function represented by an equation, write an equation that translates the graph of the function up/down, left/right.
- Given a function represented by an equation, write an equation that reflects the graph of the function across the  $x$ -axis and/or  $y$ -axis.
- Given a function represented by an equation, write an equation that expands or compresses the graph of the function horizontally and/or vertically.
- Given an equation that represents a transformation of a basic function, identify the transformation(s) involved, and graph the new function.
- Given a function represented by a table of values, and given a series of transformations to that function, identify the transformations involved, and determine the table of values for the new function.
- Given the graph of a transformation of a basic function, find the equation for that function by analyzing the transformation(s) performed, and the order of the transformations.

### Preliminaries

Given a function  $y = f(x)$ , complete the statements for the following transformation rules.

If  $c$  is a positive real number, then the graph of  $y = f(x) + c$  results from:

shifting up  $c$

If  $c$  is a positive real number, then the graph of  $y = f(x) - c$  results from:

shifting down  $c$

If  $c$  is a positive real number, then the graph of  $y = f(x + c)$  results from:

shifting left  $c$

If  $c$  is a positive real number, then the graph of  $y = f(x - c)$  results from:

shifting right  $c$

The graph of  $y = -f(x)$  results from: reflecting vertically (across  $x$ -axis)

The graph of  $y = f(-x)$  results from: reflecting horizontally (across  $y$ -axis)

If  $a$  is a positive real number, then the graph of  $y = af(x)$  results from:

stretching/compressing vertically by a factor of  $a$

When  $a > 1$ , the graph of  $y = af(x)$  is a vertical stretch

When  $0 < a < 1$ , the graph of  $y = af(x)$  is a vertical compression

If  $a$  is a positive real number, then the graph of  $y = f(ax)$  results from:

stretching/compressing horizontally by a factor of  $a$

When  $a > 1$ , the graph of  $y = f(ax)$  is a horizontal compression

When  $0 < a < 1$ , the graph of  $y = f(ax)$  is a horizontal stretch

#### Warm-up

Sketch a graph for each of the following functions.

1. The constant function  $f(x) = b$

2. The identity function  $f(x) = x$

3. The square function  $f(x) = x^2$

4. The cube function  $f(x) = x^3$

5. The absolute value function  $f(x) = |x|$

6. The square root function  $f(x) = \sqrt{x}$

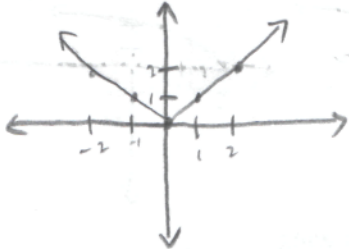
7. The cube root function  $f(x) = \sqrt[3]{x}$

8. The reciprocal function  $f(x) = \frac{1}{x}$

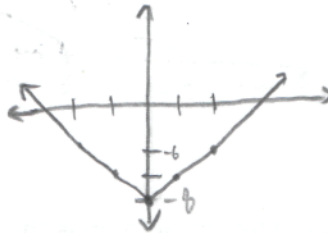
Class Notes and Examples

3.4.1 Sketch a graph of the following functions:

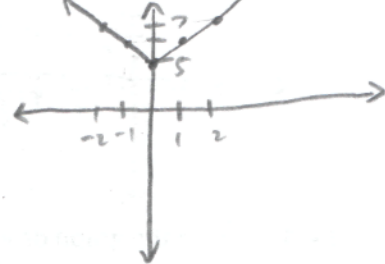
$y = |x|$



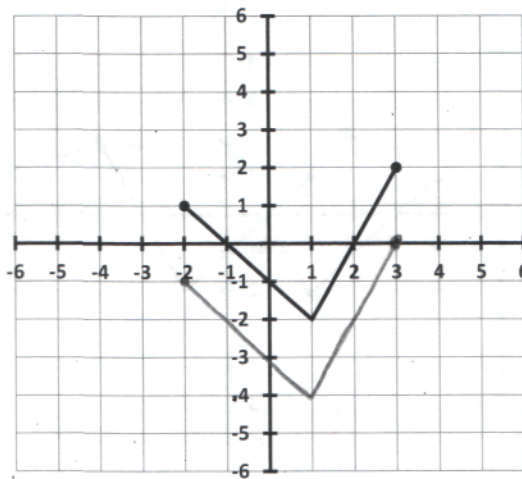
$y = |x| - 8$



$y = |x| + 5$



3.4.2 Given the graph of  $y = f(x)$  below, determine the following.



(A) Sketch the graph of  $y = f(x) - 2$  on the axes above.

(B) What are the domain and range of  $f(x)$ ?

Domain:  $[-2, 3]$

Range:  $[-2, 2]$

(C) What are the domain and range of  $f(x) - 2$ ?

Domain:  $[-2, 3]$

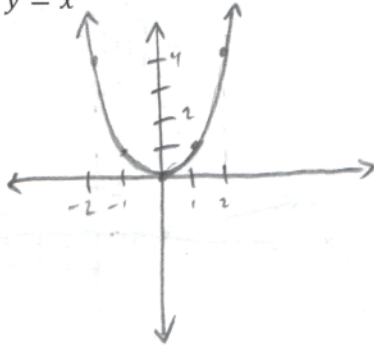
Range:  $[-4, 0]$

$$\begin{array}{ccc} \parallel & & \parallel \\ -2-2 & & 2-2 \end{array}$$

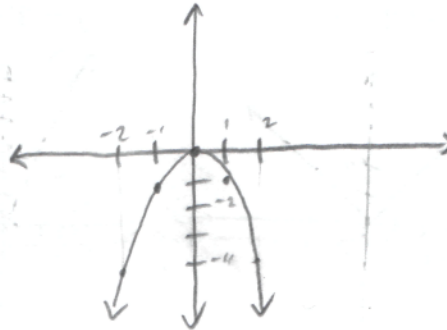


3.4.5 Sketch a graph of the following functions:

$$y = x^2$$

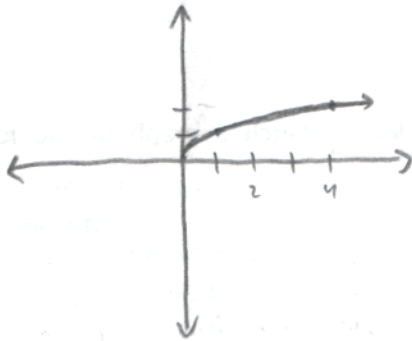


$$y = -x^2$$

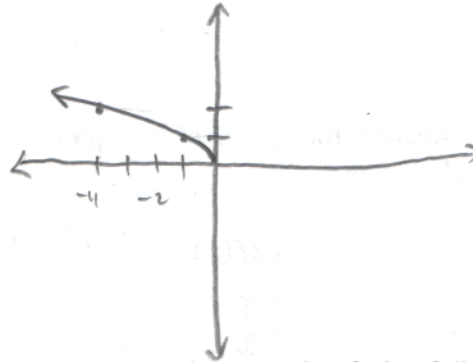


3.4.6 Sketch a graph of the following functions:

$$y = \sqrt{x}$$

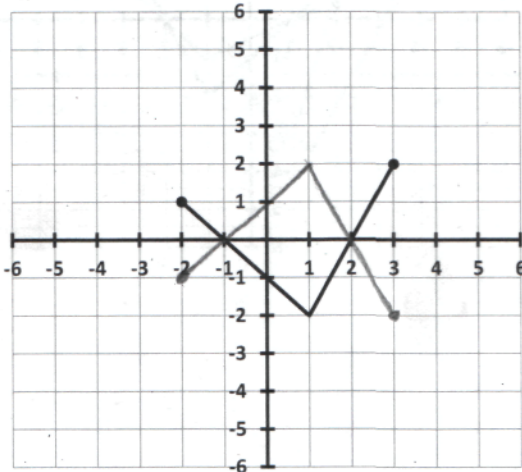


$$y = \sqrt{-x}$$

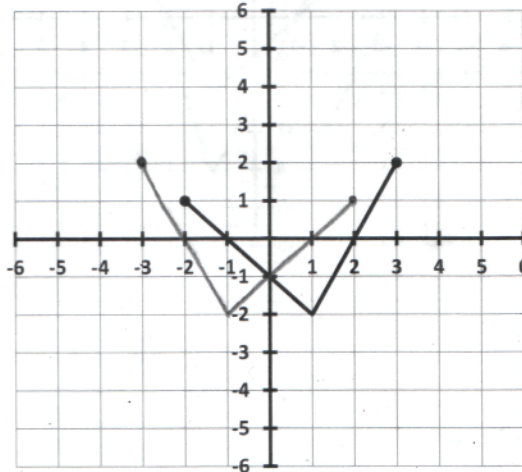


3.4.7 Assume the graph of  $y = f(x)$  is given below. Sketch a graph of the following functions:

$$y = -f(x)$$

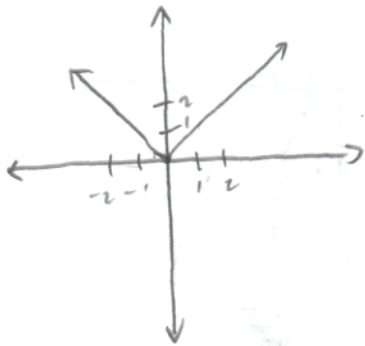


$$y = f(-x)$$

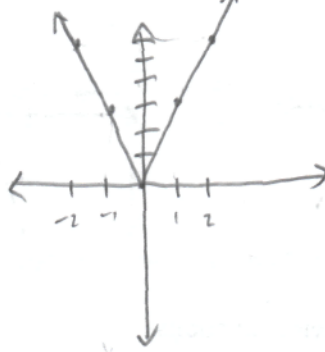


3.4.8 Sketch a graph of the following functions:

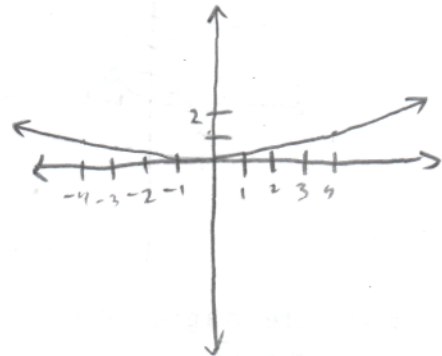
$$y = |x|$$



$$y = 3|x|$$

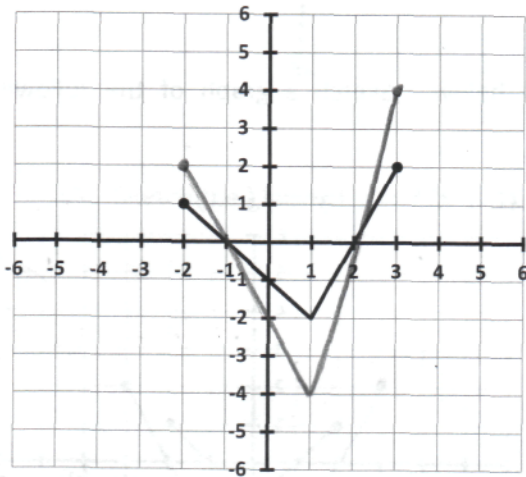


$$y = \frac{1}{4}|x|$$

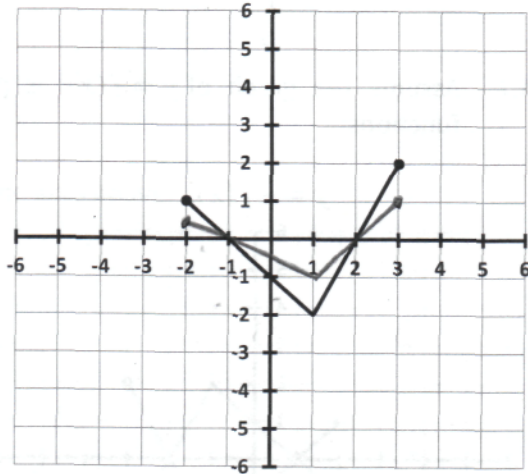


3.4.9 Assume the graph of  $y = f(x)$  is given below. Sketch a graph of the following functions:

$$y = 2f(x)$$

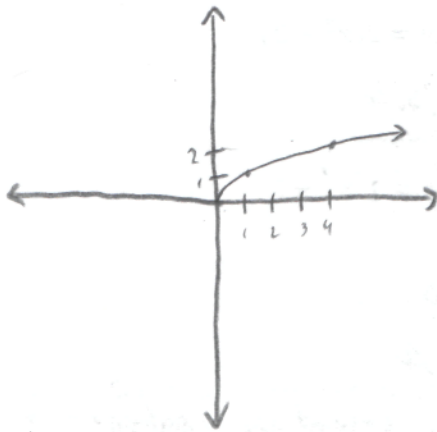


$$y = \frac{1}{2}f(x)$$

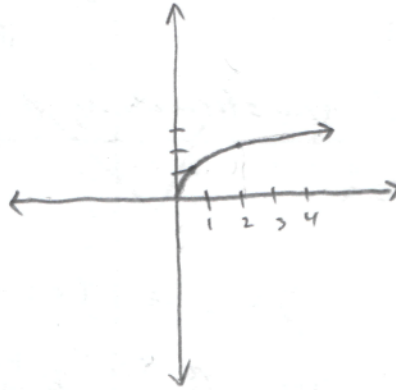


3.4.10 Sketch a graph of the following functions:

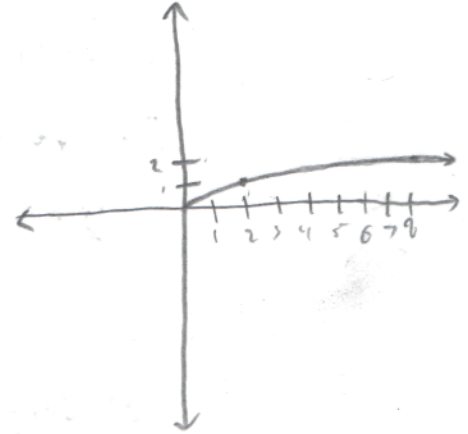
$$y = \sqrt{x}$$



$$y = \sqrt{2x}$$

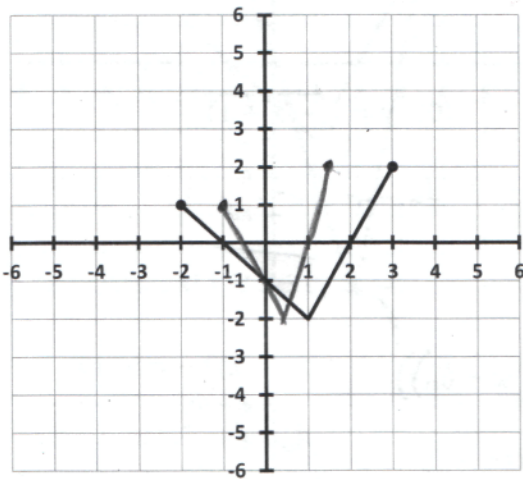


$$y = \sqrt{\frac{1}{2}x}$$

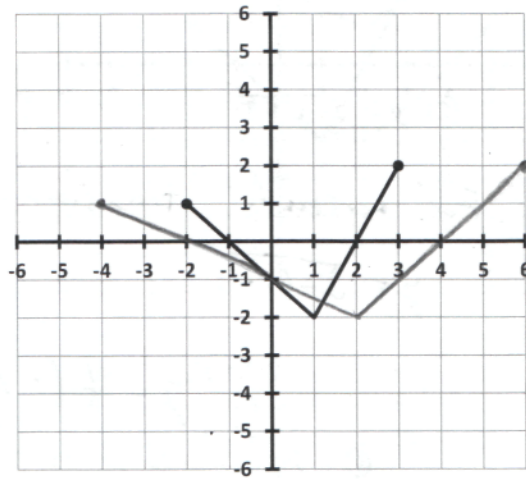


3.4.11 Assume the graph of  $y = f(x)$  is given below. Sketch a graph of the following functions:

$$y = f(2x)$$



$$y = f\left(\frac{1}{2}x\right)$$



3.4.12 Each of the functions given below have been created by performing several transformations on a base function. List the transformations performed, in order, and graph each function.

(A)  $y = 2x^2 - 3$

(B)  $y = 2(x^2 - 3)$

For vertical transformations, distribute.

$$y = 2x^2 - 3$$

$$y = 2x^2 - 6$$

$$y = \pm A f(x) + B.$$

- ① stretch/compress vertically by  $A$
- ② Reflect vertically if  $-$ . Don't reflect vertically if  $+$
- ③ shift vertically by  $B$ .

- ① stretch by a factor of 2.
- ② Down 3

- ① stretch by a factor of 2
- ② Down 6

(C)  $y = \sqrt{-x+1}$

(D)  $y = \sqrt{-(x+1)}$

For horizontal transformations, factor out.

$$y = \sqrt{-(x-1)}$$

$$y = \sqrt{-(x+1)}$$

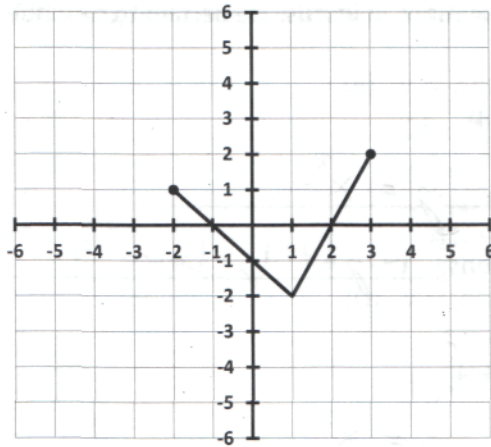
$$y = f(\pm A_0(x - B_0))$$

- ① stretch/compress horizontally by  $A_0$
- ② Reflect horizontally if  $-$ . Don't reflect horizontally if  $+$ .
- ③ shift horizontally by  $B_0$ .

- ① Reflect horizontally
- ② Right 1

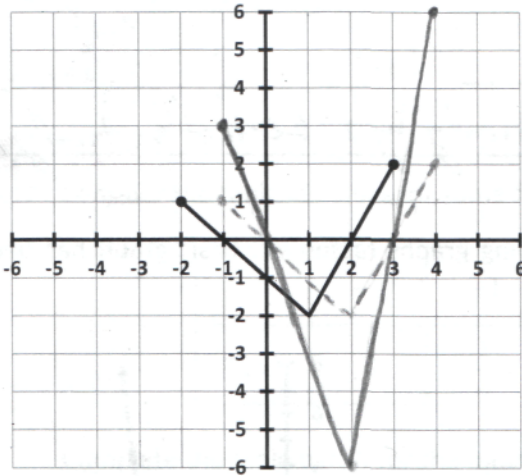
- ① Reflect horizontally
- ② Left 1

3.4.13 Let  $y = f(x)$  be given by the graph below.

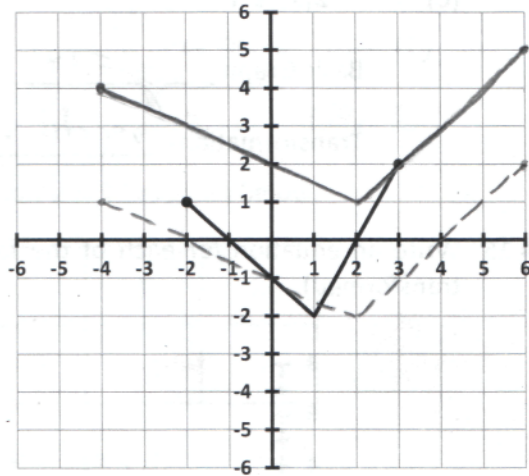


Sketch the graph of each of the following. (Note: the graph of  $y = f(x)$  has been included for reference.)

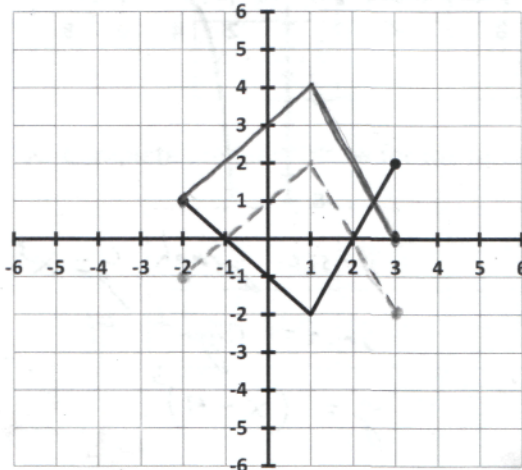
(A)  $y = 3f(x - 1)$



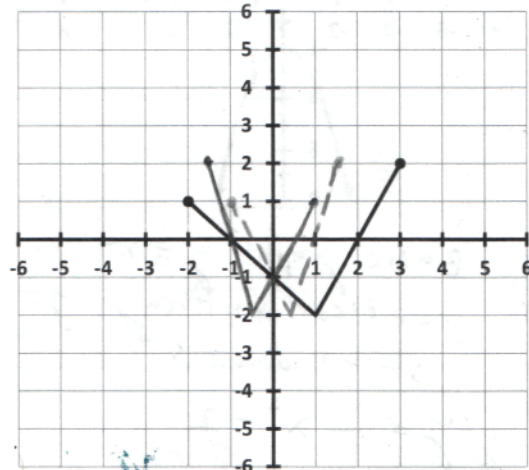
(B)  $y = f\left(\frac{1}{2}x\right) + 3$



(C)  $y = -f(x) + 2$



(D)  $y = f(-2x)$



3.4.14 For each transformation of one of our basic graphs, list the basic graph (e.g.  $y = x^2$ ) and describe the transformations in words. Be certain to describe the transformations in the appropriate order.

(A)  $y = (x - 1)^3 - 4$

Basic Graph:  $y = x^3$

Transformations: Right 1, Down 4

(B)  $y = -(x - 2)^2 + 4$

Basic Graph:  $y = x^2$

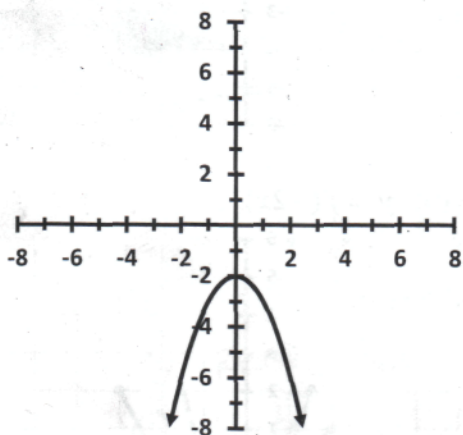
Transformations: Right 2, Reflected vertically, up 4

(C)  $y = -2|x| + 1$

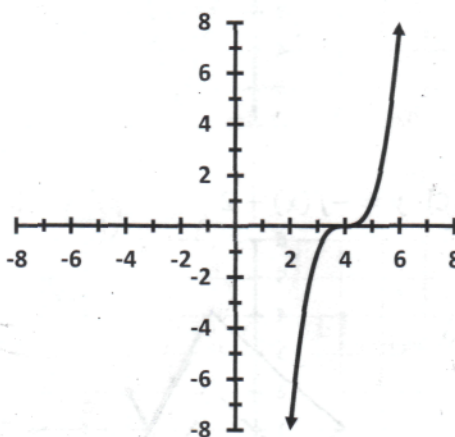
Basic Graph:  $y = |x|$

Transformations: stretch vertically by 2, Reflected vertically, up 1

3.4.15 Write an equation for each of the following graphs (assume a basic graph has been transformed).



Basic graph:  $y = x^2$   
 Reflected vertically  $y = -x^2$   
 shift down 3  
 $y = -x^2 - 2$



Basic graph  $y = x^3$   
 shift right 4  
 $y = (x - 4)^3$

3.4.16 If the point  $(2,5)$  is on the graph of  $y = g(x)$ , determine a point that MUST be on the graph of each of the following functions. Determine a way to check your answers.

(A)  $y = -g(x + 4)$

start  $(2, 5)$   
 left 4  $(-2, 5)$   
 Reflect vertically  $(-2, -5)$

(C)  $y = -3g(x) + 6$

start  $(2, 5)$   
 stretch vertically by 3  $(2, 15)$   
 Reflect vertically  $(2, -15)$   
 up 6  $(2, -9)$

(E)  $y = \frac{1}{2}g(4x)$

start  $(2, 5)$   
 compress horizontally by 4  $(\frac{1}{2}, 5)$   
 compress vertically by 2  $(\frac{1}{2}, \frac{5}{2})$

(B)  $y = 2g(-x)$

start  $(2, 5)$   
 Reflect horizontally  $(-2, 5)$   
 stretch vertically by 2  $(-2, 10)$

(D)  $y = g(x - 2) - 1$

start  $(2, 5)$   
 right 2  $(4, 5)$   
 down 1  $(4, 4)$

(F)  $y = 2g(\frac{1}{5}x) + 3$

start  $(2, 5)$   
 stretch horizontally by 5  $(10, 5)$   
 stretch vertically by 2  $(10, 10)$   
 up 3  $(10, 13)$

3.4.17 Consider the table of values for some function  $y = T(x)$  given below.

$x$	-2	-1	0	1	2
$T(x)$	3	$\frac{1}{4}$	-2	-1	6

Complete the table below for the function  $y = T(x + 3)$

$x$	-5	-4	-3	-2	-1
$T(x + 3)$	3	$\frac{1}{4}$	-2	-1	6

Complete the table below for the function  $y = -2T(x + 3)$

$x$	-5	-4	-3	-2	-1
$-2T(x + 3)$	-6	$-\frac{1}{2}$	4	2	-12