

Section 3.3

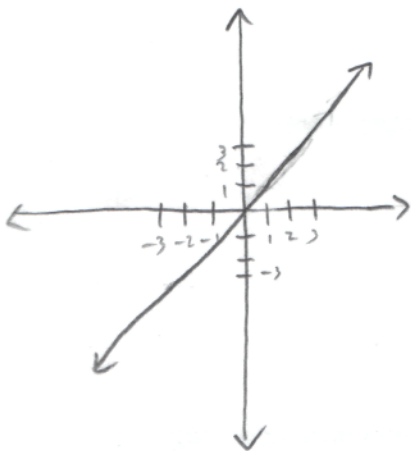
Objectives

- Identify by sight the graphs of basic functions, such as the constant function, the identity function, the square function, the cube function, the absolute value function, the square root function, the cube root function, the reciprocal function, and the greatest integer function.
- Evaluate a piecewise-defined function at a value in its domain.
- Graph a piecewise-defined function.
- Given the graph of a piecewise-defined function, find a rule that describes the function.
- Solve application problems involving piecewise-defined functions.

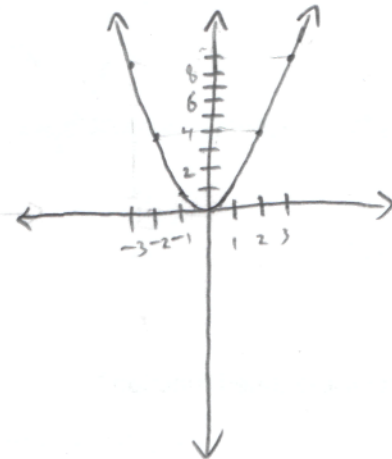
Preliminaries

Sketch the graph of each of the following functions. Indicate domain, range, and intercepts.

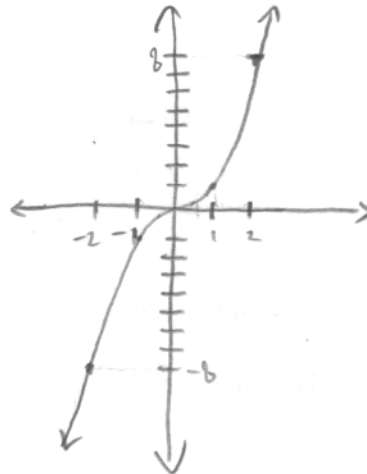
$$y = x$$



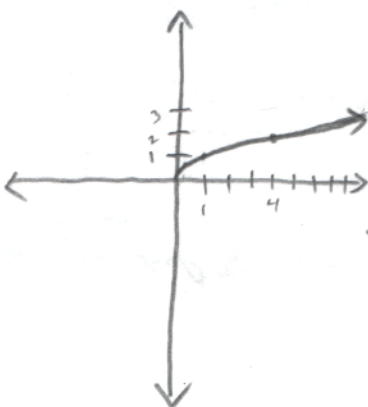
$$y = x^2$$



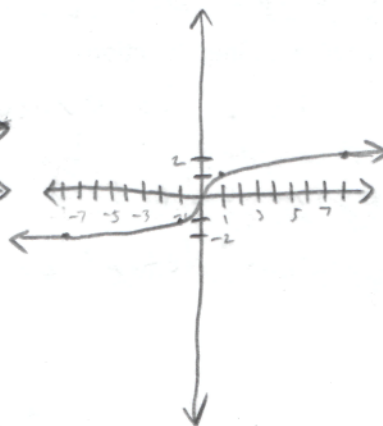
$$y = x^3$$



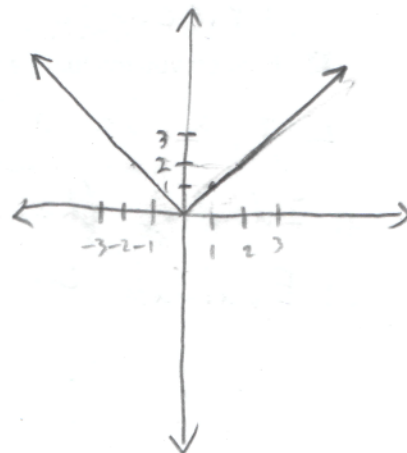
$$y = \sqrt{x}$$



$$y = \sqrt[3]{x}$$



$$y = |x|$$



Warm-up

1. Find the intercepts algebraically and graph the function: $g(x) = -2x + 5$.

2. Find the equation of the line passing through the points $(-3, -2)$ and $(-1, 5)$.

Class Notes and Examples

❓ What is meant by a piecewise-defined function?

A function defined differently on different regions.
Piecewise functions are like puzzles.

❓ How do you evaluate a piecewise-defined function?

Let "a" be a number. To evaluate $f(a)$,

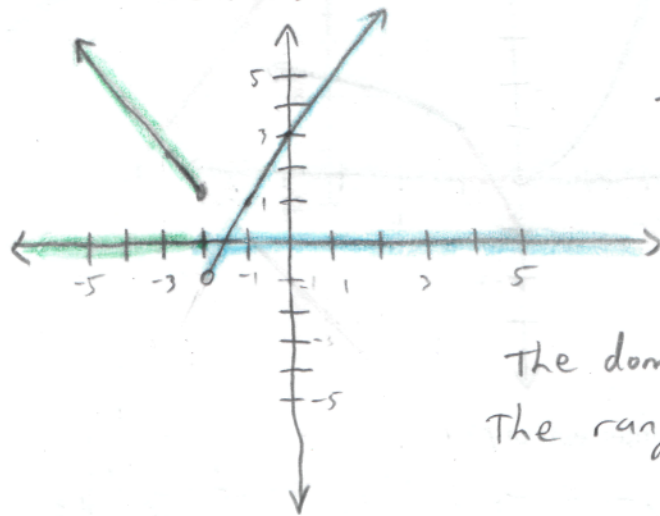
- 1) Identify the region where "a" is.
- 2) Evaluate the equation defined on that region at "a".

2 How do you graph a piecewise-defined function?

- steps:
- 1) Identify the regions on which each piece is defined.
 - 2) Graph each piece on the region it's defined.
 - a) Graph each piece for all x .
 - b) Erase the graph drawn in a) outside of the region where it's defined.

3.3.1 For the function $f(x) = \begin{cases} -x-1 & x \leq -2 \\ 2x+3 & x > -2 \end{cases}$

(A) Sketch an accurate graph of f .



Evaluate $f(-4)$, $f(1)$, and $f(-2)$.

$$f(-4) = -(-4) - 1 = 4 - 1 = 3$$

$$f(1) = 2(1) + 3 = 2 + 3 = 5$$

$$f(-2) = -(-2) - 1 = 2 - 1 = 1$$

The domain of f is $(-\infty, \infty)$
 The range of f is $(-1, \infty)$

(B) What is/are the intercept(s) of f ?

$$f(0) = 2(0) + 3 = 3. \text{ The } y\text{-intercept is } (0, 3).$$

To find x -intercepts, 1) Find the regions that have zeros.
 2) Find those zeros.

Based on the graph, f intersects the x -axis (i.e. has a zero) when $x > -2$. On that region $f(x) = 2x + 3$.

Set $f(x)$ to be zero and solve for x .

$$0 = 2x + 3$$

$$x = -3/2$$

the x -intercept is $(-3/2, 0)$.

3.3.2 Consider the piecewise function $f(x) = \begin{cases} x^2 & x < 0 \\ 2x - 1 & 0 \leq x \leq 1 \\ \sqrt{x} & x > 1 \end{cases}$

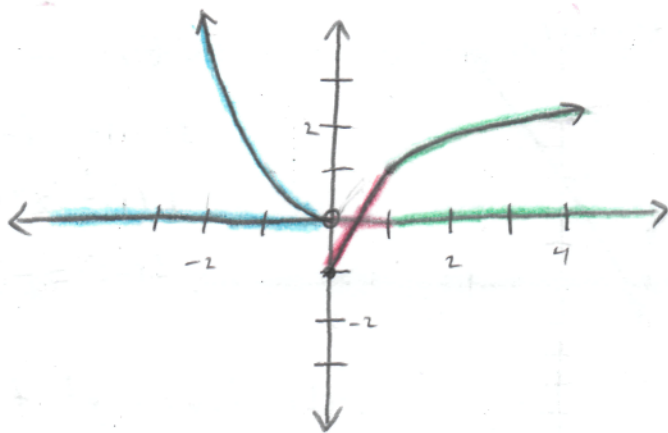
(A) Evaluate $f(-3)$, $f(1)$, and $f(5)$.

$$f(-3) = (-3)^2 = 9$$

$$f(1) = 2(1) - 1 = 2 - 1 = 1$$

$$f(5) = \sqrt{5}$$

(B) Sketch an accurate graph of f .



(C) What are the intercepts of f ?

$$f(0) = 2(0) - 1 = -1. \text{ The } y\text{-intercept is } (0, -1).$$

The x -intercept is in the region $[0, 1]$ where $f(x) = 2x - 1$.
Set $f(x)$ equal to zero and solve for x .

$$0 = 2x - 1 \quad \text{The } x\text{-intercept is } (1/2, 0).$$

$$x = 1/2.$$

(D) What is the domain of f ?

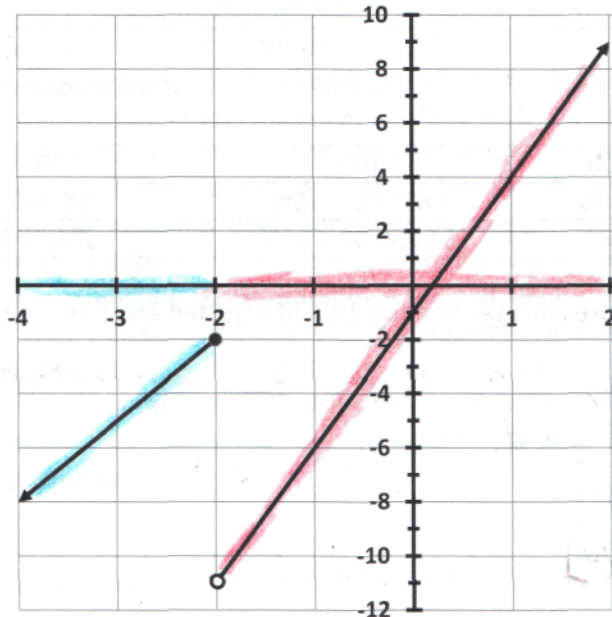
The domain of f is $(-\infty, \infty)$

(E) What is the range of f ?

The range of f is $[-1, \infty)$

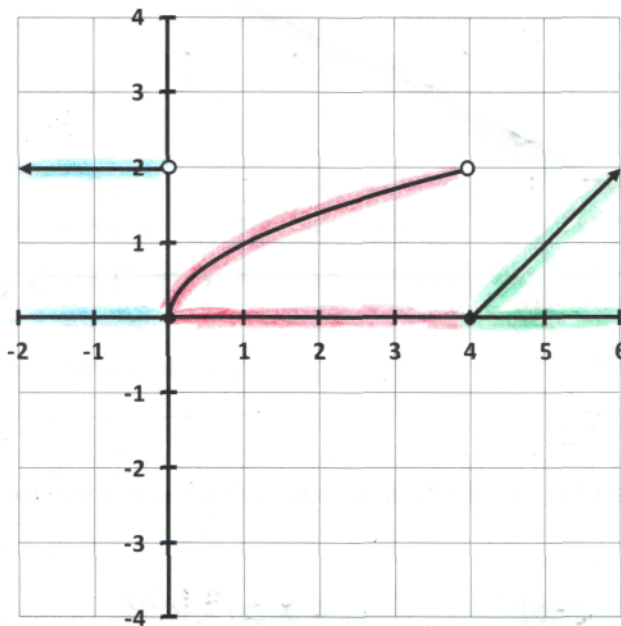
3.3.3 Give a rule for each of the functions graphed below.

(A)



$$f(x) = \begin{cases} 3x+4 & x \leq -2 \\ 0 & -2 < x < 2 \\ 5x-1 & x \geq 2 \end{cases}$$

(B)



$$f(x) = \begin{cases} 2 & x < 0 \\ \sqrt{x} & 0 \leq x < 4 \\ x-4 & x \geq 4 \end{cases}$$

- 3.3.5 The following table gives part of the Tax Rate Schedule for a person whose filing status was single during the 2012 tax year. Let's call this function $T(x)$, where x is the adjusted gross income, and $T(x)$ is the amount of tax.

Adjusted gross income is over	but not over	Tax Due	of the amount over ...
\$0	\$8,500	10%	\$0
\$8,500	\$34,500	\$850.00 + 15%	\$8,500
\$34,500	\$83,600	\$4,750.00 + 25%	\$34,500

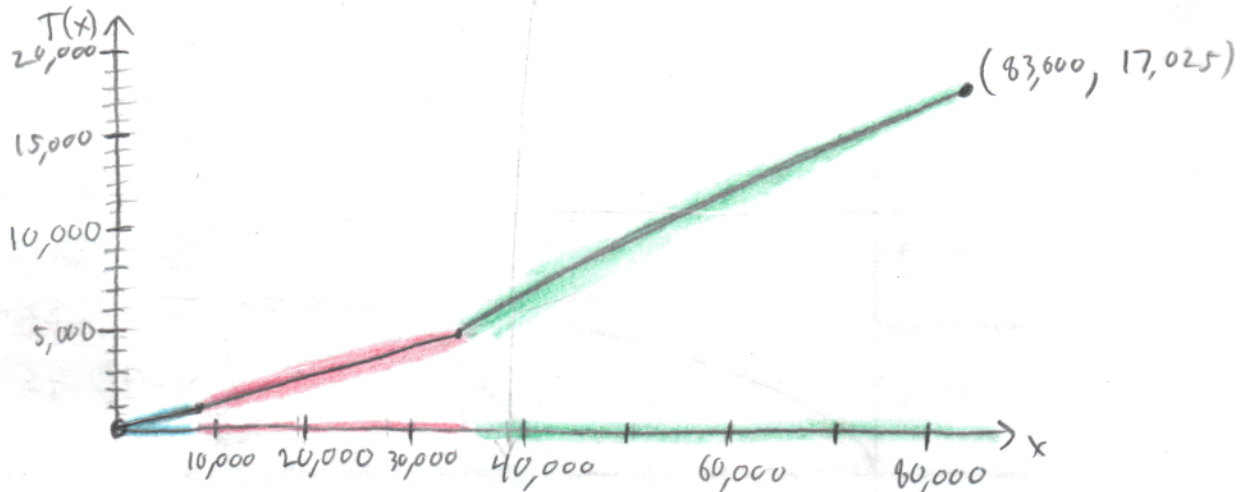
- (A) Explain why this table represents tax as a function of adjusted gross income.

For each income, there is only one amount of tax that is to be paid.

- (B) What is the domain of $T(x)$?

$$[0, 83,600]$$

- (C) Sketch an accurate graph of $T(x)$.



- (D) What is the range of $T(x)$?

$$[0, 17,025]$$

- (E) Write the rule to express tax as a function of adjusted gross income according to this table.

$$T(x) = \begin{cases} 0.10(x-0) & 0 < x \leq 8,500 \\ 850 + 0.15(x-8,500) & 8,500 < x \leq 34,500 \\ 4,750 + 0.25(x-34,500) & 34,500 < x \leq 83,600 \end{cases}$$