

Written Assignment 14 (due 4/29/13 by end of class)

Last homework assignment!

Exercise 1: Let $X = [0, 1]$ with the subspace topology from \mathbb{R} . Prove that X is not a topological manifold. (This type of space is sometimes called a *manifold with boundary* for obvious reasons).

Exercise 2: Consider the unit 2-sphere S^2 regarded as a topological manifold via stereographic projection, as done in class. Then our atlas for S^2 has two charts. Verify explicitly that S^2 with this atlas is a smooth manifold.

Exercise 3: For each of the spaces below, determine whether X is a topological manifold. If it is, also determine whether X is a smooth manifold.

- (a) $X = \{(x, y) \in \mathbb{R}^2 \mid y^2 = x^3 + x^2\}$. This is called the nodal cubic.
- (b) $X = \{(x, y) \in \mathbb{R}^2 \mid y^2 = x^3\}$. This is called the cuspidal cubic.