Written Assignment 1 (due 1/14/13 by end of class)

This first assignment is in part a means of checking everyone's background in set theory and formal logic. If the material in a question is completely unfamiliar, please let me know!

Exercise 1: Consider the statement "If Tucson is in New Mexico, then 4 is an even number".

(a) Is this statement TRUE or FALSE? Explain.

(b) State the converse of the above statement. Is it TRUE or FALSE? Explain.

(c) State the contrapositive of the original statement.

Exercise 2: Write the set $\mathbb{R} - \mathbb{Z}$ as a union of intervals.

Exercise 3: Explain the principle of induction in your own words.

Exercise 4: Construct a collection $\mathcal{A}$ of sets A such that each A is an open subset of $\mathbb{R}$ and

$$\bigcap_{A \in \mathcal{A}} A$$

is a single point.

Exercise 5: Assume that you know $\frac{d}{dx}(x) = 1$ as well as the product rule. Prove the Power Rule $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$ for positive integers n. (Differentiation doesn't really fit with the theme of this class, but the point of this problem is that induction pops up everywhere).

Exercise 6: Let $f : X \to Y$ be a function. Which of the following are always true? Either prove your answer or provide a counterexample.

(a) Let $A \subset X$. Then $f^{-1}(f(A)) = A$.

(b) Let $B \subset Y$. Then $f(f^{-1}(B)) = B$.

(c) Same statement as part (a), but you may now assume that f is injective.

(d) Same statement as part (b), but you may now assume that f is injective.

Exercise 7: State the epsilon-delta definition for a function $f : \mathbb{R} \to \mathbb{R}$ to be continuous.

Exercise 8 (Not graded): Show that your definition from the previous exercise is equivalent to the following statement. "The function $f : \mathbb{R} \to \mathbb{R}$ is continuous if for every open set A in the range, $f^{-1}(A)$ is open in the domain". I will not grade this problem, but we will see it again as one of the presentation problems for Friday 1/18/2013.

Exercise 9 (Not graded): (I will present this problem on Friday 1/11/2013 as a sample of what a presentation problem looks like.) A subset $A \subset \mathbb{R}$ is called open if, for every point $x \in A$, there is some $\epsilon > 0$ such that the interval $(x - \epsilon, x + \epsilon)$ is contained in A. The empty set $\emptyset$ is defined to be empty. Let $\mathcal{T}$ be the collection of open subsets of $\mathbb{R}$. Show that $\mathcal{T}$ satisfies the following properties.

(i) $\emptyset \in \mathcal{T}$ and $\mathbb{R} \in \mathcal{T}$

(ii) If $A_1, \ldots, A_n \in \mathcal{T}$ is a any finite set list of open sets, then $\bigcap_{i=1}^{n} A_i \in \mathcal{T}$. That is, any finite intersection of open sets is open.

(iii) If $\mathcal{A} \subset \mathcal{T}$ is an arbitrary collection of open sets (finite, countable, uncountable, whatever), then $\bigcup_{A \in \mathcal{A}} A \in \mathcal{T}$. That is, any arbitrary union of open sets in $\mathbb{R}$ is open.