1. A cone-shaped drill is drilling downward, so that the volume of ground displaced is equal to the volume of the drill in the ground. The angle from vertical to the edge of the drill is $\frac{\pi}{6}$ (picture was drawn in class, I'm skipping it here for simplicity). The drill can only move 4 cm^3 of rock per second. At the instant that the width of the hole is 10 cm, how quickly is the radius growing? Recall that a cone has volume $V = \frac{1}{3}\pi r^2 h$.

   This is a related rates problem. We are told that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. We need to get rid of one of the variables, so we use the fact that we have a $30 - 60 - 90$ triangle to say that $h = \frac{\sqrt{3}}{2} r$. Thus,

$$V = \frac{\sqrt{3}\pi}{6} \cdot r^3.$$

   Now we can differentiate both sides of the volume equation with respect to t. This yield

$$\frac{dV}{dt} = \frac{\sqrt{3}\pi}{2} \cdot r^2 \cdot \frac{dr}{dt}.$$

   Plugging in $r = 5$ and $\frac{dV}{dt} = 4$, we arrive at

$$\frac{dr}{dt} = \frac{8}{25\sqrt{3}\pi}$$

   with units of centimeters per second.

2. A river flows eastward, widening towards the ocean. For $-1 \leq x \leq 2$, the north shore is modeled by $f(x) = \frac{1}{10} \cdot e^x$, and the south shore is modeled by $g(x) = -\frac{1}{12}x^2 + \frac{1}{3}x - \frac{1}{3}$. The units are in miles.

   A toxic spill occurs at $x = -1$ and begins spreading downriver (eastward).

   (a) When the spill has reached $x = 1$, approximate the area of river affected by Riemann sums using left hand sums with $\triangle x = \frac{1}{2}$.

   The width of the rectangles is $\frac{1}{2}$, so there are 4 rectangles. Thus, the area is approximated by the left hand sum of $f(x) - g(x)$. This is equal to

   $$.5(f(-1) - g(-1)) + .5(f(-.5) - g(-.5)) + .5(f(0) - g(0)) + .5(f(.5) - g(.5)).$$

   I leave it to you to plug in the appropriate values and write down the numerical answer.

   (b) Find the exact answer to the same question using the Fundamental Theorem of Calculus.

   The area affected is equal to

$$\begin{aligned}
\int_{-1}^{1} (f(x) - g(x))\, dx &= \int_{-1}^{1} \frac{1}{10} \cdot e^x - \left(-\frac{1}{12}x^2 + \frac{1}{3}x - \frac{1}{3}\right) dx \\
&= \int_{-1}^{1} \frac{1}{10} \cdot e^x + \frac{1}{12}x^2 - \frac{1}{3}x + \frac{1}{3} dx \\
&= \left(\frac{1}{10} \cdot e^x + \frac{1}{4}x^3 - \frac{1}{6}x^2 + \frac{1}{3}x\right)\Big|_{-1}^{1} \\
&= \text{I leave the evaluation to you.}
\end{aligned}$$