# Numerical Modeling of a Fabry-Perot Laser Cavity

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December 17, 2008

## 1 Introduction

High power, ultrashort laser pulses have generated a great amount of interest in recent years. Much can be determined about the properties of these lasers by experimentation, but other phenomena can only be investigated through theory and computation. Currently there are many questions about the multimode operation of certain types of semiconductor lasers which must be addressed through numerical simulations. I will present two models of laser phenomena, a two-level atomic system with homogeneous or inhomogeneous broadening, which will aide me in constructing the actual laser model.

## 2 Model Derivation

I will present the derivation of two related models to the laser system. The first is a simple two-level atom model, followed by a generalization to an inhomogeneously broadened system. The laser system that we are considering is that of a Fabry-Perot cavity depicted in Figure 1. The cavity is 1.5 meters long with a 1.5 mm active region. The two mirrors around the cavity have reflectivities $r_1 = 1$ and $r_2 = \sqrt{0.8}$. All parameter values are listed in Table 1.

### 2.1 Homogeneously Broadened Material.

Both Fleck [1] and Yariv [2] follow essentially the same derivation procedure, with the principle difference being that Fleck derives the equations for the electric and magnetic fields simultaneously. I will be following this approach for most of the derivation. We begin with the Maxwell equations describing a plane-wave

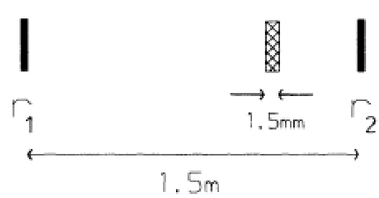


Figure 1: Schematic of the Fabry-Perot laser cavity. The two sides consist of mirrors of reflectivity $r_1 = 1$ and $r_2 = \sqrt{0.8}$. The cavity is 1.5 meters in length with a 1.5 mm active region. Parameters for the simulation are given in Table 1.

propagating in the $z$-direction

$$\frac{\epsilon}{c}\frac{\partial E_x}{\partial t} = -\frac{\partial H_y}{\partial z} - \frac{4\pi}{c}\frac{\partial P}{\partial t} - \frac{4\pi}{c}\sigma E_x \tag{1a}$$

$$\frac{\mu}{c}\frac{\partial H_y}{\partial t} = -\frac{\partial E_x}{\partial z} \tag{1b}$$

where $E$ and $H$ are the electric and magnetic fields in either the $x$- or $y$-direction, $P$ is the polarization of the material, along with material properties $\epsilon$, $\mu$ and $\sigma$ which represent the permittivity, magnetic permeability and conductivity of a dispersionless medium, respectively.

We begin with the change of variables given by

$$E^+ = \epsilon^{1/2}E_x + \mu^{1/2}H_x$$
$$E^- = \epsilon^{1/2}E_y - \mu^{1/2}H_y$$

and substituting these back into Equations (1) and set $\sigma = 0$ to simulate an electrically insulting material to get the first order system

$$\frac{\eta}{c}\frac{\partial E^+}{\partial t} + \frac{\partial E^+}{\partial z} = -\frac{4\pi\mu^{1/2}}{c}\frac{\partial P}{\partial t} \tag{2a}$$

$$\frac{\eta}{c}\frac{\partial E^-}{\partial t} - \frac{\partial E^-}{\partial z} = -\frac{4\pi\mu^{1/2}}{c}\frac{\partial P}{\partial t} \tag{2b}$$

where $\eta = \sqrt{\epsilon\mu}$ is the refractive index of the material. Equations (2) clearly describe forwards- and backwards-propagating waves coupled by their dependence on the polarization of the material.

The material is modeled as a two-level atomic system, initially as a homogeneously broadened system—that is, we do not recognize any differences between different atoms in the system–and later as an inhomogeneously broadened system. The high and low energy levels of the atoms are denoted as $E_2$ and $E_1$, along with the associated resonance frequency $\omega_0 = (E_2 - E_1)/\hbar$. In this case the density matrix becomes a simple $2 \times 2$ hermitian matrix $\rho$, where the diagonal elements $\rho_{11}$ and $\rho_{22}$ represent the probability of finding an electron in the low and high energy states, respectively, and the off-diagonal elements determine the dipole of the material.

The polarization of the material is then given by

$$P = N\bar{\mu}(\rho_{12} + \rho_{21})$$

where $N$ is the density of atoms in the material and $\bar{\mu}$ is the dipole operator parallel to the propagating fields. We can find that the differential equations satisfied by the elements of the density matrix by operations on the system Hamiltonian are

$$\frac{\partial\rho_{12}}{\partial t} + \left(\frac{1}{t_2} - i\omega_0\right)\rho_{12} = -\frac{i}{\hbar}\bar{\mu}(\rho_{11} - \rho_{22})E_x \tag{3a}$$

$$\frac{\partial(\rho_{22} - \rho_{11})}{\partial t} = \frac{1}{t_1}\left[(\rho_{22} - \rho_{11})^0 - (\rho_{22} - \rho_{11})\right] + \frac{2i}{\hbar}\bar{\mu}(\rho_{12} - \rho_{21})E_x \tag{3b}$$

where $t_1$ and $t_2$ are the transverse and longitudinal relaxation times and $(\rho_{22} - \rho_{11})^0$ is the steady-state difference in the populations of the atomic levels.

We can introduce a complex polarization $P_c = N\bar{\mu}\rho_{12}$ and note that $P = P_c + P_c^*$. Further, the population inversion will be defined as $n = N(\rho_{22} - \rho_{11})$. We can see that, by adding the complex conjugate to both sides of equation (3a) that the right hand side is identically zero. Then we have

$$\frac{\partial P}{\partial t} = \left(\frac{1}{t_2} - i\omega_0\right)P_c + \left(\frac{1}{t_2} + i\omega_0\right)P_c^*.$$

Follow this with the change of variables

$$E^{\pm'} = \sqrt{\frac{c}{16\pi\eta\hbar\omega_0}} E^{\pm}$$

$$P' = i\frac{4\pi\omega_0}{c}\sqrt{\frac{\mu c}{16\pi\eta\hbar\omega_0}} P_c$$

and we can rewrite Equations (2) and (3) as

$$\frac{\eta}{c}\frac{\partial E^{+'}}{\partial t} + \frac{\partial E^{'+}}{\partial t} = -(1+i\delta)P' + (1-i\delta)P'^{*} \tag{4a}$$

$$\frac{\eta}{c}\frac{\partial E^{-'}}{\partial t} - \frac{\partial E^{-'}}{\partial t} = -(1+i\delta)P' + (1-i\delta)P'^{*} \tag{4b}$$

$$\frac{\partial P'}{\partial t} + (\frac{1}{t_2} - i\omega_0)P' = -\frac{1}{2t_2}\sigma_0 n(E^{+} + E^{-}) \tag{4c}$$

$$\frac{\partial n}{\partial t} + \frac{1}{t_1}(n - n^0) = 4(P' + P'^{*})(E^{+} + E^{-}) \tag{4d}$$

where $\delta = (\omega_0 t_2)^{-1}$ and $\sigma_0 = (4\pi\bar{\mu}\omega_0 t_2/(\hbar c))\sqrt{\mu/\epsilon}$.

## 2.2 Inhomogeneously Broadened Material

Unlike in the previous construction, an inhomogeneous broadening acknowledges that there are differences between the atoms of the system. I will use the superscript $\xi$ as an index to differentiate the different type of atoms possible.

Atoms of the same "family" are considered to have energy levels $E_2^{\xi}$ and $E_1^{\xi}$, and more importantly, the same resonant angular frequency, $\omega^{\xi} = (E_2^{\xi} - E_1^{\xi})/\hbar$. We describe the proportion of atoms with the same resonant frequency by a probability model, where the probability of an atom having resonant angular frequency between $\omega^{\xi}$ and $\omega^{\xi} + d\omega^{\xi}$ is given by $p(\omega^{\xi})d\omega^{\xi}$. Then we can change equations (3) into the larger system

$$\frac{\partial \rho_{12}^{\xi}}{\partial t} + \left(\frac{1}{t_2} - i\omega^{\xi}\right)\rho_{12}^{\xi} = -\frac{i}{\hbar}\bar{\mu}(\rho_{11}^{\xi} - \rho_{22}^{\xi})E_x \tag{5a}$$

$$\frac{\partial(\rho_{22}^{\xi} - \rho_{11}^{\xi})}{\partial t} = \frac{1}{t_1}\left[(\rho_{22}^{\xi} - \rho_{11}^{\xi})_0 - (\rho_{22}^{\xi} - \rho_{11}^{\xi})\right] + \frac{2i}{\hbar}\bar{\mu}(\rho_{12}^{\xi} - \rho_{21}^{\xi})E_x \tag{5b}$$

where the density matrix $\rho$ must also be indexed with an element $\xi$.

In order to reach the state of equations (4) we must define some "total effect" terms by

$$\overline{P_c} = \int_{\mathbb{R}} P_c^{\xi}(\omega)p(\omega^{\xi})d\omega^{\xi} \tag{6a}$$

$$\omega_0 = \int_{\mathbb{R}} \omega^{\xi} p(\omega^{\xi})d\omega^{\xi} \tag{6b}$$

where $P_c^{\xi}(\omega) = N\bar{\mu}\rho_{12}^{\xi}$, similar to that defined in the homogeneously broadened system.

We make the same change of variables as in the homogeneously broadened case and end up with the set of equations

$$\frac{\eta}{c}\frac{\partial E^{+'}}{\partial t} + \frac{\partial E^{+'}}{\partial t} = -(1+i\delta)\overline{P'} + (1-i\delta)\overline{P'}^* \tag{7a}$$

$$\frac{\eta}{c}\frac{\partial E^{-'}}{\partial t} - \frac{\partial E^{-'}}{\partial t} = -(1+i\delta)\overline{P'} + (1-i\delta)\overline{P'}^* \tag{7b}$$

$$\frac{\partial P'^{\xi}}{\partial t} + (\frac{1}{t_2} - i\omega^{\xi})P'^{\xi} = -\frac{1}{2t_2}\sigma_0 n(E^+ + E^-) \tag{7c}$$

$$\frac{\partial n^{\xi}}{\partial t} + \frac{1}{t_1}(n^{\xi} - n^{\xi,0}) = 4(P'^{\xi} + P'^{\xi *})(E^+ + E^-) \tag{7d}$$

where

$$\overline{P'} = \int_{\mathbb{R}} P'^{\xi}(\omega)p(\omega^{\xi})d\omega^{\xi}$$

### 2.3 Envelope Approximation and Fourier Series Approximation to form Transverse Wave Equations

To simplify matters, we make the slowly-varying envelope approximation by the system

$$E^{+'}(z,t) = \mathcal{E}^+(z,t)e^{i(\omega_0 t - kz)} + \mathcal{E}^{+*}e^{-i(\omega t - kz)} \tag{8a}$$

$$E^{-'}(z,t) = \mathcal{E}^-(z,t)e^{i(\omega_0 t - kz)} + \mathcal{E}^{-*}e^{-i(\omega t - kz)} \tag{8b}$$

$$P'(z,t) = \rho(z,t)e^{i\omega_0 t} \tag{8c}$$

and additionally assume that $\omega_0 t_2$ is large, so $\delta \approx 0$. Note that the reference frequency of the system has explicitly been chosen as the operating frequency of the laser, that is, the frequency inside the envelope. In the inhomogeneous case the reference frequency will be chosen as the expected frequency of the ensemble of atoms. Then plugging this into equations (7) and averaging over may rapid spatial variations gives

$$\frac{\eta}{c}\frac{\partial \mathcal{E}^+}{\partial t} + \frac{\partial \mathcal{E}^+}{\partial z} = -\langle \rho e^{ikz}\rangle \tag{9a}$$

$$\frac{\eta}{c}\frac{\partial \mathcal{E}^-}{\partial t} - \frac{\partial \mathcal{E}^-}{\partial z} = -\langle \rho e^{ikz}\rangle \tag{9b}$$

$$\frac{\partial \rho}{\partial t} + \frac{\rho}{t_2} = -\frac{\sigma_0 n}{2t_2}\left(\mathcal{E}^+ e^{-ikz} + \mathcal{E}^- e^{ikz}\right) \tag{9c}$$

$$\frac{\partial n}{\partial t} + \frac{n - n^0}{t_1} = 4\left(\rho\mathcal{E}^{+*}e^{ikz} + \rho\mathcal{E}^{-*}e^{-ikz} + \text{c.c.}\right). \tag{9d}$$

The exponential terms are then dealt with by use of the Fourier series expansions

$$\rho(z,t) = e^{-ikz}\sum_{p=0}^{\infty}\rho_p^+ e^{-2ipkz} + e^{ikz}\sum_{p=0}^{\infty}\rho_p^- e^{2ipkz}$$

$$n(z,t) = n_0 + \sum_{p=1}^{\infty}(n_p e^{-2ipkz} + \text{c.c.})$$

Substituting the above into equations (4) gives the system

$$\frac{\eta}{c}\frac{\partial \mathcal{E}^{\pm}}{\partial t} \pm \frac{\partial \mathcal{E}^{\pm}}{\partial t} = -\rho_0^{\pm} \tag{10a}$$

$$\frac{\partial \rho_p^{+}}{\partial t} + \frac{1}{t_2}\rho_p^{+} = -\frac{\sigma_0}{2t_2}\left(n_p\mathcal{E}^{+} + n_{p+1}\mathcal{E}^{-}\right) \tag{10b}$$

$$\frac{\partial \rho_p^{-}}{\partial t} + \frac{1}{t_2}\rho_p^{-} = -\frac{\sigma_0}{2t_2}\left(n_p^{*}\mathcal{E}^{-} + n_{p+1}^{*}\mathcal{E}^{+}\right) \tag{10c}$$

$$\frac{\partial n_{p+1}}{\partial t} + \frac{1}{t_1}n_{p+1} = 4\left(\rho_p^{+}\mathcal{E}^{-*} + \rho_p^{+}\mathcal{E}^{+*} + \rho_p^{-*}\mathcal{E}^{+} + \rho_{p+1}^{-*}\mathcal{E}^{-}\right) \tag{10d}$$

$$\frac{\partial n_0}{\partial t} + \frac{1}{t_1}(n - n^0) = 4\left(\mathcal{E}^{-*}\rho_0^{-} + \mathcal{E}^{+*}\rho_0^{+} + \text{c.c.}\right) \tag{10e}$$

for $p = 0, 1, 2, \ldots$.

I will assume that the lowest order Fourier term is dominant, as in [3, 4]. This simplifies equations (10) to the system

$$\frac{\eta}{c}\frac{\partial \mathcal{E}^{\pm}}{\partial t} \pm \frac{\partial \mathcal{E}^{\pm}}{\partial t} = -\rho_0^{\pm} \tag{11a}$$

$$\frac{\partial \rho_0^{\pm}}{\partial t} + \frac{1}{t_2}\rho_0^{\pm} = -\frac{\sigma_0}{2t_2}n_0\mathcal{E}^{\pm} \tag{11b}$$

$$\frac{\partial n_0}{\partial t} + \frac{1}{t_1}(n_0 - n_0^0) = 4\left(\mathcal{E}^{-*}\rho_0^{-} + \mathcal{E}^{+*}\rho_0^{+} + \text{c.c.}\right) \tag{11c}$$

The derivation to the inhomogeneously broadened system is very similar, where the equations (11) becomes

$$\frac{\eta}{c}\frac{\partial \mathcal{E}^{\pm}}{\partial t} \pm \frac{\partial \mathcal{E}^{\pm}}{\partial t} = -\rho_0^{\pm} \tag{12a}$$

$$\frac{\partial \rho_0^{\xi\pm}}{\partial t} + \left(\frac{1}{t_2} + i(\omega_0 - \omega^{\xi})\right)\rho_0^{\xi\pm} = -\frac{\sigma_0}{2t_2}n_0^{\xi}\mathcal{E}^{\pm} \tag{12b}$$

$$\frac{\partial n_0^{\xi}}{\partial t} + \frac{1}{t_1}(n_0^{\xi} - n_0^{\xi,0}) = 4\left(\mathcal{E}^{-*}\rho_0^{\xi-} + \mathcal{E}^{+*}\rho_0^{\xi+} + \text{c.c.}\right) \tag{12c}$$

Notice that by assuming only first-order Fourier effects we have pushed the coupling back another level in both equations (11) and (12). That is, $\mathcal{E}^{\pm}$ and $\rho_0^{\pm}$ interact with each other, and the $+$ and $-$ terms are now only coupled through the population inversion.

The final steps in constructing the differential equations studied is to renormalize the variables and make the time and space dimensions unitless. The time and space units are normalized by the width of the active medium, so $z = Lz'$ and $t = (\eta L/c)t'$. With this applied the simulation variables are renormalized as

$$E^{\pm} = \frac{\bar{\mu}}{2}\sqrt{\frac{16\pi\eta\omega_0 t_1 t_2}{c\hbar}}E^{\pm\prime}$$

$$P = \frac{2}{\sigma_0\sqrt{\mu}}\sqrt{\frac{16\pi\eta\omega_0 t_1 t_2}{c\hbar}}\frac{L\omega\bar{\mu}^2 t_2}{2c\epsilon\hbar}P'$$

$$D = \frac{L\omega\bar{\mu}^2 t_2}{2\eta c\epsilon\hbar}n,$$

where I have returned to the original notation for the electric field and polarization for simplicity. Then

| Parameter | value | Parameter | value |
|---|---|---|---|
| $L_{cav}$ | 1.5 m | $L$ | 1.5 mm |
| $r_1$ | 1.0 | $r_2$ | $\sqrt{0.8}$ |
| $N$ | $3.0 \times 10^{18}$ cm$^{-3}$ | $\sigma$ | $2.0 \times 10^{-17}$ cm$^2$ |
| $\eta$ | 1.47 | $t_1$ | 1.6 $\mu$s |
| $t_2$ | 2 ps | $A_s$ | $1.0 \times 10^{-6}$ |
| $t_p$ | $\approx$ 10 ns | $t_{FWHM}$ | 80 ps |

Table 1: Parameter values for the laser model

applying all of this gives the final governing equations of the model

$$\pm\frac{\partial E^{\pm}}{\partial z'} + \frac{\partial E^{\pm}}{\partial t'} = -P^{\pm}, \tag{13a}$$

$$\frac{\partial P^{\pm}}{\partial t'} = -\Gamma_2\left(P^{\pm} + E^{\pm}D\right) + S^{\pm}, \tag{13b}$$

$$\frac{\partial D}{\partial t'} = -\Gamma_1\left[D - A(t) - \frac{1}{2}\left(E^{+*}P^{+} + E^{-*}P^{-} + \text{c.c.}\right)\right] \tag{13c}$$

where $\Gamma_1$ and $\Gamma_2$ are the phenomenalogical damping constants for $D$ and $P^{\pm}$, are given by

$$\Gamma_1 = \frac{\eta L}{c}\frac{1}{t_1} \tag{14a}$$

$$\Gamma_2 = \frac{\eta L}{c}\frac{1}{t_2} \tag{14b}$$

where $L$ is the length of the lasing medium. Important parameter values are given in Table 1.

There was one further change in the system of equations, replacing $D^0$ with the function $A(t)$. Lasers must be pumped in order to gain the power required to lase. Pulsing lasers must be pumped non-uniformly in time in order to create their pulses. The function $A(t)$ is used to establish a time-varying population inversion which will all for laser operation by using it to represent a train of Gaussian pulses of width $t_{FWHM}$ coming into the laser.

The inhomogeneously broadened case must be treated differently. Having a single pumping function $A(t)$ will cause all atoms to be pumped equally, which is non-physical for distributions of atoms which are wider than the spectrum of the pump pulse. The governing equations for the inhomogeneous system then are

$$\pm\frac{\partial E^{\pm}}{\partial z'} + \frac{\partial E^{\pm}}{\partial t'} = -P^{\pm}, \tag{15a}$$

$$\frac{\partial P_{\xi}^{\pm}}{\partial t'} = -\Gamma_{2,\xi}\left(P_{\xi}^{\pm} + E^{\pm}D_{\xi}\right) + S_{\xi}^{\pm}, \tag{15b}$$

$$\frac{\partial D_{\xi}}{\partial t'} = -\Gamma_1\left[D_{\xi} - A(t) - \frac{1}{2}\left(E^{+*}P_{\xi}^{+} + E^{-*}P_{\xi}^{-} + \text{c.c.}\right)\right]. \tag{15c}$$

## 2.4 Pumping in Inhomogenously Broadened System

The system of equations 15 does not allow for the different atoms to be correctly pumped. In a physical system the atoms will or will not be pumped depending on where their intrinsic frequency falls within the spectrum of the pump pulse. System 15 will cause all atoms to be pumped equally, and thus be non-physical. As a result we replaced $A(t)$ by $D_0^0$, the steady-state population-inversion, and tried other methods of pumping inhomogeneous system.

### 2.5 Boundary Conditions

The equations in (4) and (7) are designed to simulate how a laser pulse evolves inside a cavity. Specifically, a Fabry-Perot cavity is used, which has two mirrors and light propagating in both directions between them. Then, if $z = z_1$ and $z = z_2$ are the positions of the two mirrors, the boundary conditions are

$$E^+(z_1, t) = -r_1 E^-(z_1, t)$$
$$E^-(z_2, t) = -r_2 E^+(z_2, t)$$

where $r_1$ and $r_2$ are the reflectivities of the mirrors at $z_1$ and $z_2$.

Additionally, the equations for the polarization and population are only valid within the active region of the laser, which only takes up a small portion of the total volume. In the remainder of the cavity the laser is described by the pair of transport equations for $E^\pm$ without any forcing term.

## 3 Numerical Scheme

### 3.1 Equations Represented

As observed in [1] there are two types of equations that must be solved numerically. The first type is given by

$$\frac{dx}{dt} + Tx = f$$

where $x$ is some place-holder variable. We can use an integrating factor to integrate from $t_m = m\Delta t$ to $t_{m+1}$ and get the form

$$x_j^{m+1} = x_j^m e^{-T\Delta t} + \int_{t_m}^{t_{m+1}} e^{-(t-t')T} f(t')dt'$$

and the using a variant of the trapezoid rule (i.e., assuming that $f$ is linear in this region)

$$x_j^{m+1} = Cx_j^m + Af_j^m + Bf_j^{m+1} + O(\Delta t^2) \tag{16}$$

where

$$C = e^{-\Delta t T} \tag{17a}$$

$$A = \frac{1}{T}\left[\frac{1}{T\Delta t}\left(1 - e^{-T\Delta t}\right) - e^{-T\Delta t}\right] \tag{17b}$$

$$B = \frac{1}{T}\left[1 - \frac{1}{T\Delta t}\left(1 - e^{-T\Delta t}\right)\right] \tag{17c}$$

The second type of equation is given in the form

$$\pm\frac{\partial y}{\partial z} + \frac{\partial y}{\partial t} = g$$

which is integrated along the characteristic lines to get

$$y_j^{n+1} - y_{j\mp1}^n = \frac{1}{2}\Delta z\left(g_j^{n+1} + g_{j\mp1}^n\right) + O(\Delta z^3) \tag{18}$$

### 3.2 Specifics of implementation

When the above development is applied the specific equations given in Forysiak, we get the equations for the electric fields as

$$E_j^{+,n+1} - E_{j-1}^{+,n} = -\frac{1}{2}\Delta z \left(P_j^{+,n+1} + P_{j-1}^{+,n}\right) \tag{19a}$$

$$E_j^{-,n+1} - E_{j+1}^{-,n} = -\frac{1}{2}\Delta z \left(P_j^{-,n+1} + P_{j+1}^{-,n}\right) \tag{19b}$$

the equations for the polarization are

$$P_j^{+,n+1} = C_2 P_j^{+,n} - A_2\left(\Gamma_2 E_j^{+,n} D_j^n\right) - B_2\left(\Gamma_2 E_j^{+,n+1} D_j^{n+1}\right) + A_s e^{i\phi_j^{n+1}} \tag{20a}$$

$$P_j^{-,n+1} = C_2 P_j^{-,n} - A_2\left(\Gamma_2 E_j^{-,n} D_j^n\right) - B_2\left(\Gamma_2 E_j^{-,n+1} D_j^{n+1}\right) + A_s e^{i\phi_j^{n+1}} \tag{20b}$$

and finally for the population inversion

$$\begin{aligned} D_j^{n+1} = C_1 D_j^n &+ A_1\Gamma_1\left[A^n + \frac{1}{2}\left(E_j^{+,n} P_j^{+,n*} + E_j^{-,n} P_j^{-,n*} + \text{c.c.}\right)\right] \\ &+ B_1\Gamma_1\left[A^{n+1} + \frac{1}{2}\left(E_j^{+,n+1} P_j^{+,n+1*} + E_j^{-,n+1} P_j^{-,n+1*} + \text{c.c.}\right)\right] \end{aligned} \tag{21}$$

with the above parameters are taken from (17) and translate into

$$C_{1,2} = e^{-\Gamma_{1,2}\Delta t'} \tag{22a}$$

$$A_{1,2} = \frac{1}{\Gamma_{1,2}}\left[\frac{1}{\Gamma_{1,2}\Delta t'}\left(1 - e^{-\Gamma_{1,2}\Delta t'}\right) - e^{-\Gamma_{1,2}\Delta t'}\right] \tag{22b}$$

$$B_{1,2} = \frac{1}{\Gamma_{1,2}}\left[1 - \frac{1}{\Gamma_{1,2}\Delta t'}\left(1 - e^{-\Gamma_{1,2}\Delta t'}\right)\right] \tag{22c}$$

Of course these equations are only valid within the lasing medium. Outside of this medium the propagating fields are handled by array-pointer systems.

Fleck suggested a predictor-corrector type method for solving the implicit system of equations given by (19), (20) and (21), which is outlined as

1. Extrapolate the inversion as $D_j^{n+1} = 2D_j^n - D_j^{n-1}$. This is possible because the inversion changes on a much slower time scale than either the field or the polarization.
2. Use this extrapolation to solve equations (19) and (20) simultaneously for $E^{\pm}$ and $P^{\pm}$.
3. Use these estimates to recompute $D_j^{n+1}$.

## 4 Results

The first goal of this project was to replicate the results that Forysiak and Moloney produced in [3]. The differential system was implemented in a computer and the behavior of the system confirmed with the previous results. The evolution of the pulse over time can be seen in Figure 2. The plots show the intensity of the forward propagating electric field ($|E^+|^2$) and the population inversion as measured at one end of the active medium over a single cavity round trip at various points in the evolution of the pulse. The measure of time is centered at the the maximum of the pump pulse. Note the rapidly changing scales of the light intensity over time and that the laser pulse lags behind the pump pulse somewhat. We can see that the population inversion is constant for some time after the pump pulse. This shows the time required for the pulse to propagate from the active medium to the outcoupling mirror and back.

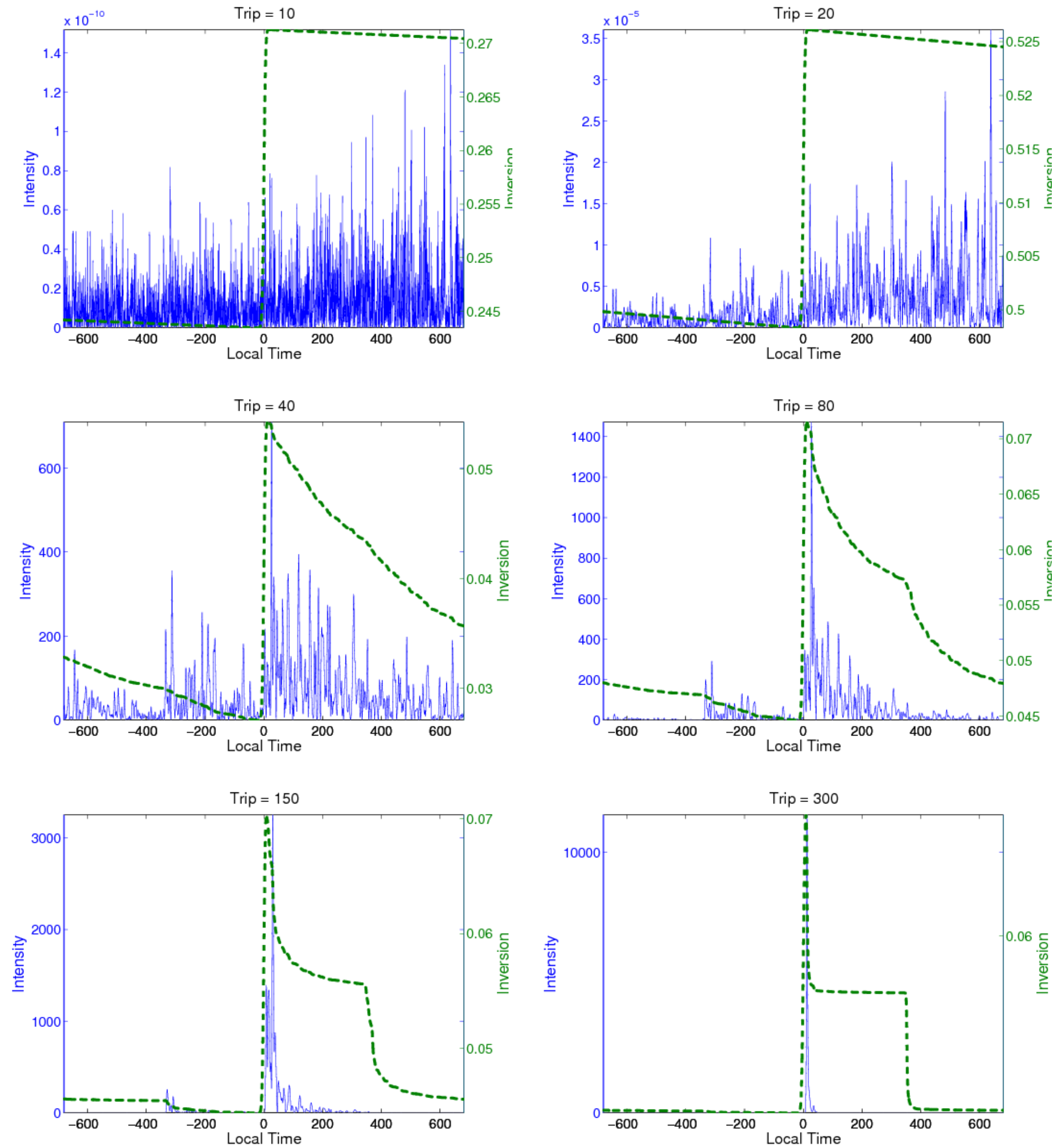


Figure 2: Plots demonstrating the evolution of a pulse over the first few hundred round trips. The left hand axis shows the intensity of the light propagating in one direction along the Fabry-Perot laser cavity, $|E^+|^2$, sampled at one end of the active medium over the course of one cavity round trip. The right hand axis shows the population inversion $D$ sampled at the same point. The abscissa is centered at the time when the pump pulse reaches its maximum. Early time plots show the pulse being built up from simulated noise in responce to the pump pulse. We can see that the laser pulse lags behind the pump pulse somewhat. The flat plateau seen in the plot at 300 round trips (and developing in the previous 3 plots) shows the time from the pulse needs to travel from the active medium to the outcoupling mirror and back again. The pulse larger pulse is favored over the smaller one due to the active medium not being centered within the cavity.

## 5 Future Work

As of the end of this project the inhomogeneously broadened system is not well studied. There is still much to be investigated about this. In particular, I was unable to create an adequate way to pump the inhomogeneously broadened system. This would be the first correction needed in a continuation of this project.

Once the inhomogeneous model has been thoroughly tested the free carrier model of a material will be implemented. This model expands on the inhomogeneity within the two-level atom model and presents more realistic results for a wider variety of lasing materials. Upon implementing this model the laser will be switched to a Vertical External Cavity Surface Emitting Laser (VECSEL) rather than a color-center laser. VECSELs are of high interest in the field and are capable of creating significantly shorter pulses than color center lasers.

Finally, the results of the free carrier model will be fed into an existing code based on the many-body model. This model is more realistic still but much slower computationally, so running an entire simulation with it is impractical. The build-up of the laser pulse is to be done in the simpler free carrier model, while a detailed study of the results will be done in the many-body model.

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