On the motion of a single bead of viscous fluid down a vertical wire

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Outline

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Initial Observations

- A variety of interesting behaviors were observed in the initial videos
  - Bead formation
  - Coalescence
  - Beads of various sizes moving along the wire
- The motion of individual beads was chosen as subject of study
  - Clearly observable relationship between bead volume and velocity
  - Similar to observable everyday phenomena, such as raindrops on a window
  - Applicable to industrial processes such as vapor condensation on a cold tube, inkjet printing

Project Description

Experimental Objective

To experimentally verify the observed volume/velocity relationship.

Theoretical Objective

To propose a simple mechanical model, in hopes that it would lend insight into the experimental results.

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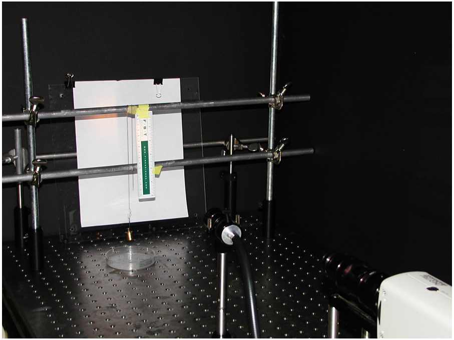
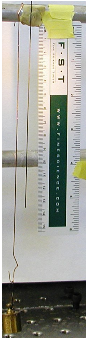
Experimental Setup





Materials

- Clover honey
- Hardened, straightened piano wire - 0.37mm and 0.63mm

Experimental Challenges

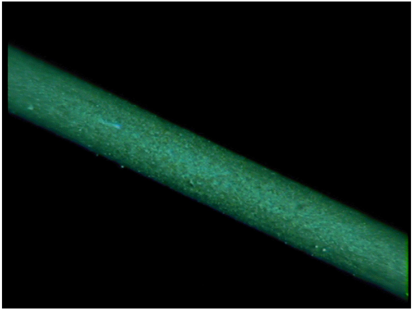


Figure: 0.63mm piano wire under 4x zoom

- Piano was chosen as it exhibited few irregularities.
- Copper wire was also tried, but it had visible deformations due to its manufacturing process.

Procedure

- Honey added to the wire through a syringe
  - Some control on the size of the bead
  - Bead did not necessarily remain on one side of wire
- Video recording taken of each experiment
- Several measurements obtained using an image editing program
  - Ellipse fitted to bead
  - Volume of resultant ellipsoid calculated
  - Estimate of instantaneous velocity found using calculated center of mass

Data by Bead for 0.37mm Wire

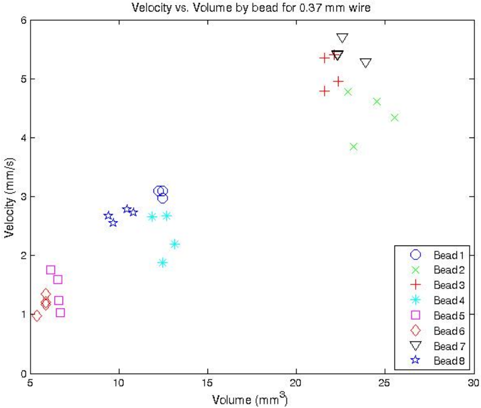
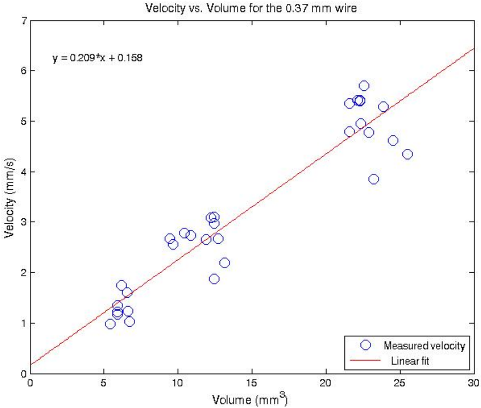


Figure: Velocity vs. volume for 0.37mm wire

Data for 0.37mm Wire



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Data for 0.63mm Wire

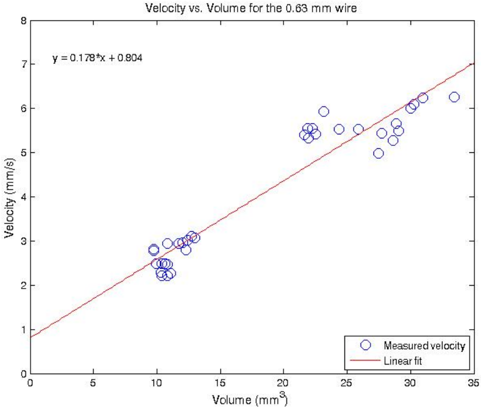

Figure: Velocity vs. volume for 0.63mm wire

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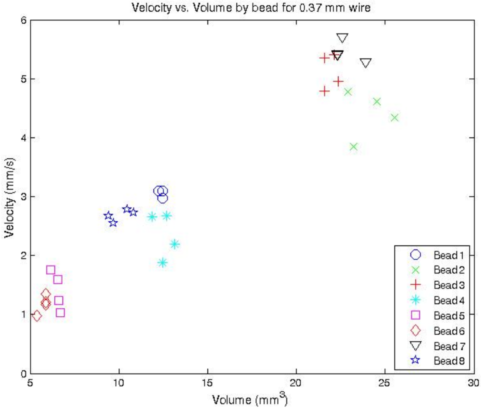

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Statistical Analysis

Two linear models were fitted to each experiment to determine if there is a non-negligible change in bead volume and velocity:

$$volume = \alpha_1 + \beta_1 * time \quad \text{and} \quad velocity = \alpha_2 + \beta_2 * time$$

The null hypotheses for these models are:

$$H_0 : \beta_1 = 0 \quad \text{and} \quad H_0 : \beta_2 = 0$$

- 0.37mm wire: no statistically significant values of β_1 and β_2
- 0.63mm wire: a few statistically significant values, with $\beta_1 < 0$ and $\beta_2 < 0$

Conclusions

- Differing diameter wires may change bead behavior
- Possibility of a slight decrease in volume and velocity during a single experiment
- However, this change could not be measured to within a reasonable degree of accuracy
- Hence, can assume that volume and velocity do not change during an experiment for both wires

Conclusion

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1. Introduction
2. Experiments
3. Rigid Body Approximation
   - Theoretical Model
   - Results
4. Conclusions

Theoretical Model

Goal

Test the efficacy of a simple physical model at predicting the motion of the bead.

Propose

$$m\ddot{x} = mg - F(\dot{x}, m)$$

x: bead position	m: bead mass
$\dot{x}$: bead velocity	g: gravitational acceleration
$\ddot{x}$: bead acceleration	F: friction force

Friction Force Model

Assumptions:

- Form of a power law with respect to velocity
- Mass dependence embedded in "terminal velocity" function

$$F(\dot{x}, m) = \alpha\,(\dot{x})^{\gamma}$$

At terminal velocity (v_T), i.e., when $\ddot{x} = 0$, equation reduces to

$$F(v_T(m)) = \alpha\,(v_T(m))^{\gamma} = mg$$

Equation of motion

$$\ddot{x} = g\left(1 - \left(\frac{\dot{x}}{v_T(m)}\right)^{\gamma}\right)$$

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Parameters

The terminal velocity profile is taken from the experimental data, i.e.,

$$v_T(m) = \begin{cases} 0.209\frac{m}{\rho} + 0.158 & 0.37 \text{ mm wire} \\ 0.178\frac{m}{\rho} + 0.804 & 0.63 \text{ mm wire} \end{cases}$$

where ρ is the density of honey (m/ρ is the estimated volume).

Power law parameter γ was to be fitted to the recorded data in the least squares sense. Some parameters

- x_i^0, initial position of the *i*th bead
- $\dot{x}_i^0$, initial velocity of the *i*th bead
- t_i^0, time until first measurement for th *i*th bead

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Least Squares

The Matlab program `lsqcurvefit` was used to fit the parameter γ and the unknown experimental parameters. The set of ODEs was solved using `ode15s` with the initial conditions

$$\begin{cases} x(0) & = -x_i^0 \\ \dot{x}(0) & = 0 \end{cases}$$

The resulting curves were fitted to the experimental data.

Rigid Body Results I

Rigid body model was compared to experimental results

Parameter	0.37 mm wire	0.63 mm wire
γ range:	1.5 - 2.15	0.9 - 1
x_i^0 range (cm):	0.3 - 1.0	0.3 - 1.0
t_i^0 range (s):	1 - 2	1 - 2

- Optimization seems to have found that γ depends on the wire size.
- Lower bound of initial position is smaller than in experiment, where beads were deposited at least 1 cm above camera viewing range.

Rigid Body Results II

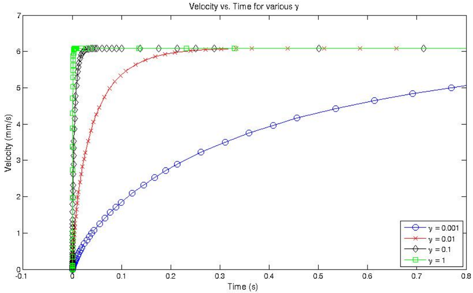
The search space was restricted to only allow for beads that began at least 1 cm above the camera viewing range.

Result:

- Bead initial position was between 1.5 and 2 cm
- Time to first measurement was 2 - 10 s
- γ increased to 3.0 for 0.37 mm wire, 2.5 for 0.63 mm wire

The initial position and time results were closer to experimental conditions.

Role of γ?



- We believed that the γ dependence on wire size was caused by changes in rates of acceleration
- However, the above plot shows that acceleration is nearly instantaneous for $\gamma > 0.1$
  - All beads had reached terminal velocity by the time the first measurement was taken.

Conclusions

- Reliance on fitted parameters ensures that the mechanical model presented here does not allow for predictions of physical phenomena.
- The friction model has some shortcomings
    - No method of predicting changes in terminal velocity as a function of mass
    - A sliding friction is independent of surface area. Predictions of surface area effects will require fluid dynamic calculations

Outline

Conclusions

- Velocity of a bead depends roughly linearly upon its size
- Velocity and volume of a bead did not change significantly over the course of a single experiment
- Rigid body model inadequate as formulated to describe the volume/velocity relationship

Future Work I

- Comparing motion on a clean wire to motion on a pre-wetted wire
  - Available literature suggests governing dynamics are different.
  - However, basic size/velocity relationship should be similar.
  - Experiments could be conducted to determine differences in this relationship.
- Examining loss of mass from bead as it travels down the wire
  - A bead leaves behind a thin film as it travels.
  - Decreasing mass should correlate with a decrease in velocity.
  - It is difficult to quantify the change in either variable for a single bead using current experimental setup.

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Future Work II

- Determining the motion of the fluid within the bead, and how this relates to overall motion
  - We inadvertently observed a circular, “rolling” motion within the bead.
  - This could relate to how fluid is deposited on the solid surface as bead moves

Acknowledgments

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