On the motion of a single bead of viscous fluid moving down a vertical wire

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Abstract

This paper present the results of a series of experiments designed to measure the relationship between the volume and velocity of single bead of viscous fluid (represented by honey) moving down a vertical wire. Wires of two different diameters were used, as well as beads of differing sizes. A simple model was constructed using the assumption of rigid body motion as an attempt to provide understanding of the experimental results.

1 Introduction

The dynamics of bead motion on a planar or cylindrical solid surface is a subject of interest due to its presence in a variety of commonly observed phenomena, such as raindrops sliding down a window. A complete understanding of this problem has clear applications to industrial processes such as ink-jet printing and the condensation of vapor on a cold tube surface [1].

Honey was chosen as the viscous fluid of interest as it is readily accessible. A variety of interesting behaviors were observed when the fluid was applied to a thin wire, including the formation of beads from a thin coating, bead coalescence, and the movement or lack thereof of individual beads. One particular subset of these behaviors, namely, the motion of an individual bead on the wire and the relationship of this motion to the volume of the bead, was selected as the subject of this project. Preliminary observations suggest that, as the mass of the bead increases relative to the surface area of the honey/wire interface, the velocity of the bead will increase. Conversely, if this ratio is sufficiently low, the forces causing the bead to adhere to the wire will overcome inertial effects and the bead will be motionless.

In previous work, Goren [2] discusses the dynamics of bead formation, determining that the spacing of the beads formed from a thin coating results from inherent instabilities. Hattori et al [3] continue this theme by considering a constant flow down a wire and examining the effects of wire diameter on the instability. Quéré [4] examines the conditions necessary to preserve a strictly annular flow. These papers consider the problem from the point of view of a continuous flow of fluid. However, the dynamics are considerably different in the case of a single bead moving along a solid surface in the absence of any preexisting coating of fluid. In traditional hydrodynamic theory, a singularity arises when attempting to describe the behavior of the advancing contact line, i.e., the intersection of the fluid, solid, and gas interfaces. A review of this topic can be found in [5].

In this paper, the authors aim to provide experimental verification of the observed dependence between bead volume and velocity. In addition, this paper briefly examines whether or not a proposed simple mechanical model can be used to provide insight into the experimental results.

2 Experiments

2.1 Experimental Materials

Mass produced clover honey, specifically Safeway brand, was used as a typical representative of a viscous fluid in this experiment. The viscosity of honey varies quite substantially with both temperature and water content, changing from 138 P to 20.4 P as water content increases from 15.5% to 20.2% at room temperature [6]. The honey used for the experiment was found to have density 1.5 g/mL and viscosity 85 P determined by the method described in [7].

Hardened, straightened piano wire was chosen as the vertical wire for the experiments. Two diameters of piano wire, 0.37 mm and 0.63 mm, were selected for experimental use in order to ascertain the effect of increased surface contact area on the motion of the bead. The piano wire did not exhibit particular irregularities, as confirmed by viewing both types of wire under a standard upright microscope with a 4x zoom lens.

2.2 Experimental Methodology

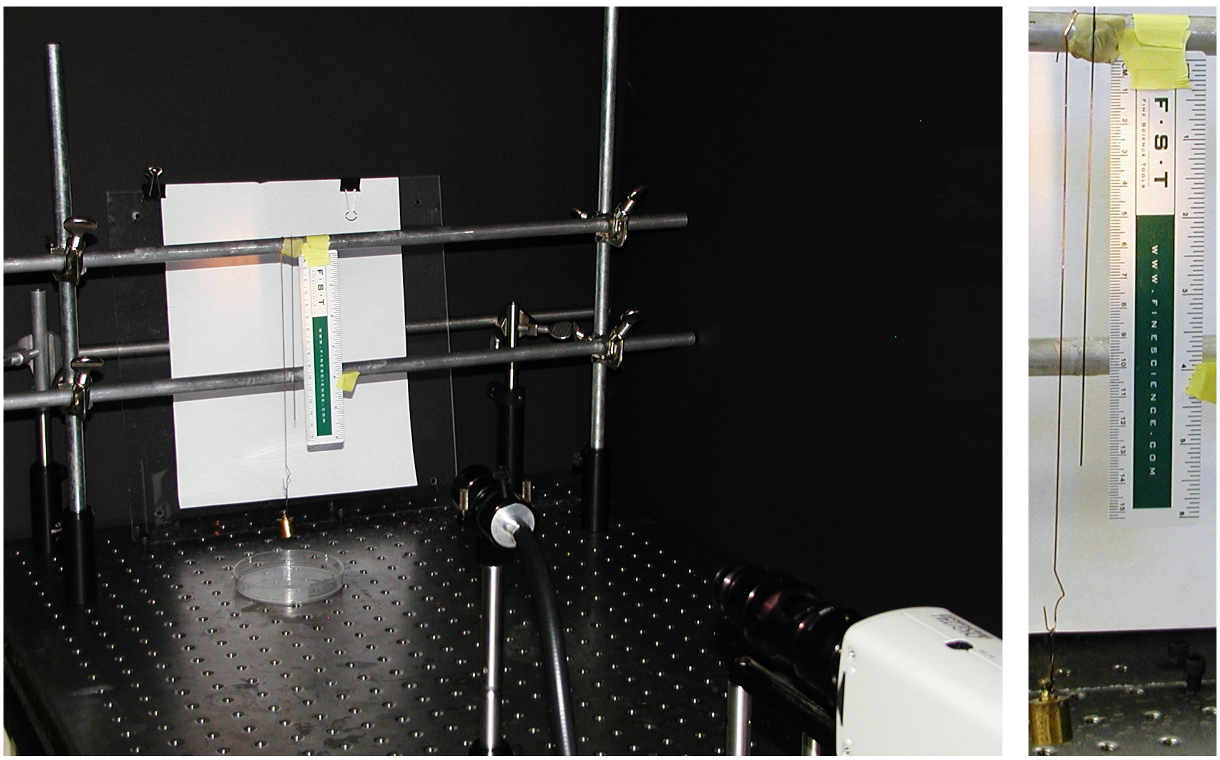
The experimental setup is shown in Figure (1). The piano wire was suspended between two horizontal bars. To ensure that any oscillations would be quickly damped, it was connected to the top bar and positioned to contact the lower bar. A second wire was also suspended from the top bar, and a weight was attached at the end to ensure that the wire was vertical. Prior to beginning each experiment, the vertical suspension of the piano wire was confirmed by checking that the horizontal distance between it and the guide wire was constant throughout the visual range of the camera. A weight was not secured to the piano wire so that no bends were induced in the already straight wire.

Honey was added consistently to a single side of the wire via a blunted, 16 gauge syringe needle. However, the resultant bead did not necessarily remain on that side as the experiment progressed. The honey was added several centimeters above the view of the camera to ensure that the edge effects associated with the deposition had a limited effect on later behavior. This process allowed a coarse level of control over the size of the bead that was to be deposited. No deposition method was found that allowed for fine control over the bead size, due to the tendency of the honey to stick together and to resist transfer from the syringe to the wire. It was hoped that random variation in bead size would help to create a more even distribution of bead sizes. This technique had only limited success, see Figures (2a) and (2b).

The wire was cleaned in a two-step process between each experiment. First, the wire was removed from the mounting and wiped with a paper towel. It was then cleaned with Ethyl Alcohol to remove any honey remaining and allowed to dry. The piano wire was handled only with latex gloves to prevent any contamination with oils from skin.

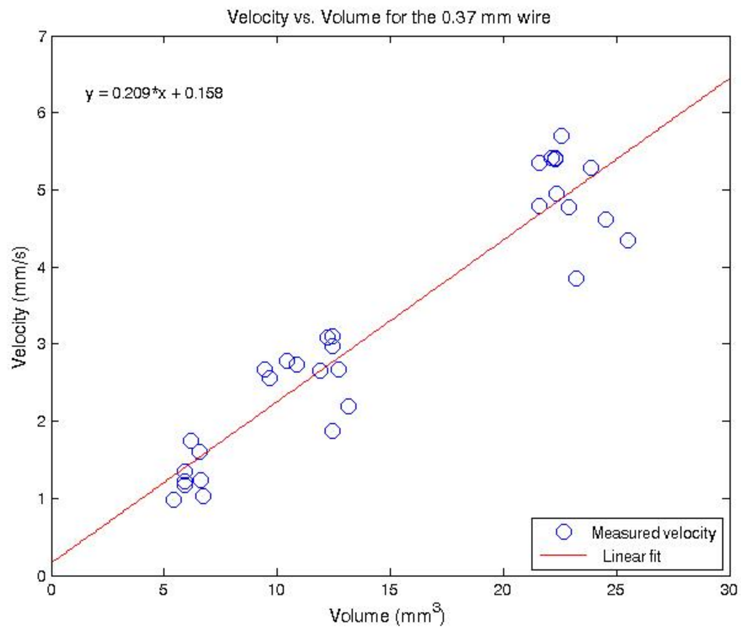
A video recording of each experiment was taken using a color video camera [8]. The image was captured simultaneously on VHS videotape and in digital format [9]. The digital video was recorded and stored on a computer through the use of a commercial software package [10]. Another piece of software [11] was used to convert the video into a format suitable for analysis.

Measurements of bead size were obtained by visually estimating the perimeter of the bead under study and then fitting an ellipse that matched that perimeter using a tool available in an image analysis software package [12]. This was done at several distinct points in each video. The dimensions of this ellipse at each point was then used to calculate the volume of an ellipsoid (taking the polar radius as an estimate for the smaller equatorial radius) to give an approximation for that bead's volume at that time. The software package also reported the center of mass of the fitted ellipse. The displacement of this center over time as the video progressed was recorded and used to calculate the average velocity of the bead over several intervals in each video.

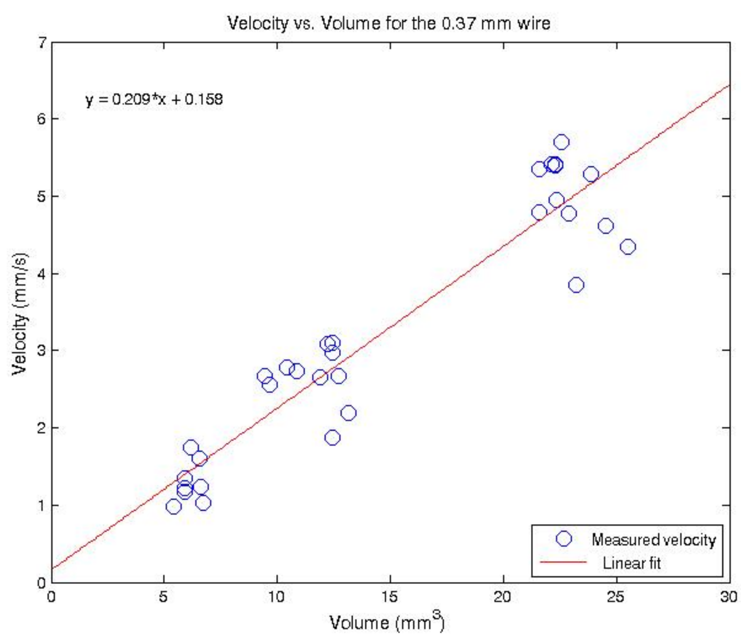


(a) (b)

Figure 1: Photographs of the experimental setup, in wide angle (1a) and closeup (1b)



(a)



(b)

Figure 2: Velocity vs. bead volume for the 0.37 mm wire (2a) and 0.63 mm wire (2b)

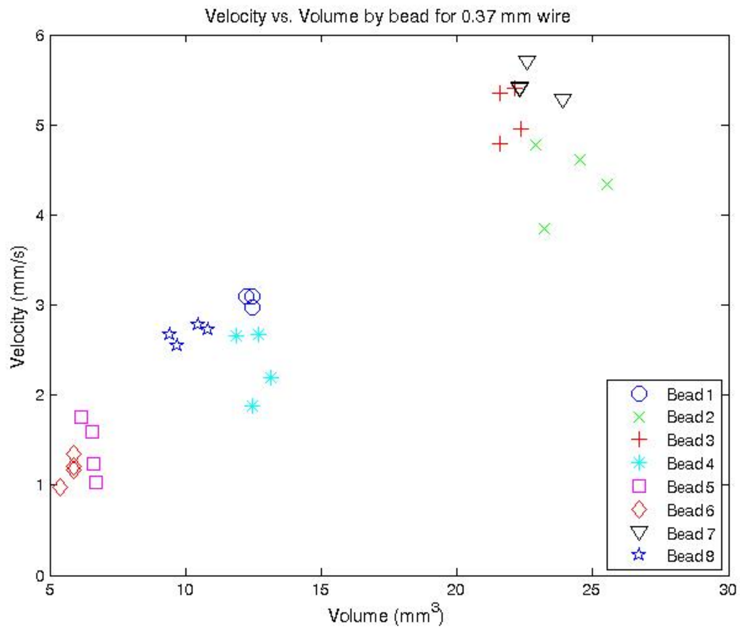


Figure 3: 0.37 mm wire by bead

2.3 Experimental Results

Figures (2a) and (2b) present the results obtained by the experiments described above. Both graphs contain all the data points given from that particular wire size, and a least squares trendline has been fitted to the data. Figure (3) groups the data points from the 0.37 mm wire by experiment. This graph and a similar one for the 0.63 mm wire indicate that, although there is variation in bead volume and velocity within an experiment, the overall differences between experiments are not solely a result of fluctuations within experiments. However, it is reasonable to predict that the volume and velocity will change as an experiment progresses since the bead leaves a trail of honey behind it as it moves down the wire.

To determine if there is a non-negligible change in the evolution of the volume and velocity over time of a bead, two linear models giving volume and velocity as a function of time were fitted for each experiment, i.e.

$$volume = \alpha_1 + \beta_1 * time \qquad \text{and} \qquad velocity = \alpha_2 + \beta_2 * time$$

The null hypotheses for the models are

$$H_0 : \beta_1 = 0 \qquad \text{and} \qquad H_0 : \beta_2 = 0$$

respectively. Tables 1 and 2 show the resultant p-values for both models; these give the probability of obtaining values of β_1 and β_2 under the null hypothesis for repeated experiments. For the 0.37 mm wire, none of the p-values are significant (all are greater than 0.05), so it is reasonable to assume that within one experiment, the volume and velocity of the bead are constant. The 0.63 mm wire has a few significant p-values; these are represented in boldface in Table 2. The corresponding values of β_1 and β_2 are negative, indicating that both volume and velocity decrease with time, as expected. That the 0.63 mm wire produced significant p-values, while the 0.37 mm wire did not, demonstrates that it's possible that beads moving down larger diameter wires lose a higher proportion of mass. However, although there may be a slight decrease in volume and velocity in time, assuming that these parameters do not change over the course of an experiment is justifiable for both sizes of wires.

Therefore, it is plausible to say that there is indeed a relationship between volume and velocity, and that velocity increases as volume increases.

Bead	Volume	Velocity
1	0.3499	0.3513
2	0.1785	0.8646
3	0.2684	0.2684
4	0.5967	0.815
5	0.4466	0.2922
6	0.2927	0.7357
7	0.3490	0.3153
8	0.1226	0.7858

Table 1: p-values for the 0.37 mm wire

Bead	Volume	Velocity
1	0.6666	0.5225
2	**0.0157**	**0.0325**
3	0.0666	**0.0385**
4	0.9145	0.8218
5	**0.0260**	**0.0004**
6	0.4022	0.2372
7	0.2720	0.5225
8	0.3142	0.2547

Table 2: p-values for the 0.63 mm wire

3 Rigid Body Approximation

3.1 Theoretical Model

Due to the complexities inherent in the system described above, it is very difficult to use standard hydrodynamic theory to provide a mathematical description of the behavior of a honey bead on a wire. Hence, in this section, the authors aim to construct a simplified mechanical model that still captures the observed relationship between the volume of a bead and its velocity.

The model is as follows:

$$m\ddot{x} = mg - F(\dot{x}, m)$$

where m is the mass of the bead, assumed to be constant in light of the above statistical results, x is its position on the wire, g is the gravitational acceleration and $F(x)$ is the "friction" force between the bead and the wire, which is assumed to vary with both bead velocity $\dot{x}$ and mass. Experimental data is used to provide an approximate relationship between the terminal velocity of the bead v_T and its mass. At terminal velocity, the acceleration of the bead $\ddot{x}$ vanishes, leaving the simplified equation $F(v_T(m), m) = mg$. The model for the friction is constructed by making the *a priori* assumption that friction is a power law with respect to the terminal velocity, and that the mass dependence is described entirely by the terminal velocity function. These assumptions yield

$$F(v_T, m) = \alpha \left(v_T(m)\right)^{\gamma} = mg$$

Clearly, $\alpha = \frac{mg}{v_T(m)^{\gamma}}$, so the generalized friction function becomes

$$F(\dot{x}, m) = mg\left(\frac{\dot{x}}{v_T(m)}\right)^{\gamma}$$

By varying the size of the bead deposited on the wire in experiment, an estimate of the terminal velocity of the bead as a function of the approximate volume was obtained (see Figures (2a) and (2b)), i.e., the function

$$v_T(m) = \begin{cases} 0.209\frac{m}{\rho} + 0.158 & 0.37 \text{ mm wire} \\ 0.178\frac{m}{\rho} + 0.804 & 0.63 \text{ mm wire} \end{cases}$$

was used, where ρ is the density of honey and $\frac{m}{\rho}$ is the estimated volume. The power law coefficient γ is determined through the non-linear least squares optimization procedure `lsqcurvefit` within Matlab to fit the experimental model to the measured results. Some additional parameters are required for the rigid body model that were not given by the experimental results: the initial position of the bead relative to the viewing range of the camera, the initial velocity and the time it took for the bead to become visible to the camera. The initial velocity is assumed to be zero in the model, and additional fitting parameters x_i^0 and t_i^0 were introduced to the optimization routine to estimate the initial position and time required to enter the field of view of the camera for the ith bead.

For each bead, the above ordinary differential equation was solved numerically in Matlab subject to the initial conditions $x(0) = -x_i^0, \dot{x}(0) = 0$ as described above. The experimental measurement times were adjusted so that the first measurement was taken at time t_i^0, i.e., the first measurement was taken when the bead first entered the sight of the camera. The difference between the experimental data and theoretical model was then minimized in the least-squares sense by varying γ and the parameters x_i^0 and t_i^0.

3.2 Results

Fitting the models to experimental data from both the 0.37 mm and 0.63 mm wires revealed that the power law coefficient γ depends on the size of the wire being measured. Least squares fitting found several different optimized values of γ for the case of differing wire diameters: for the 0.63 mm diameter wire, γ was consistently between 0.9 and 1, but γ was in the range of 1.5 - 2.15 for the 0.37 mm wire.

Examination of the fitted parameters for the time from release to the time of the first measurement and the distance between the initial position and the position of the first measurement reveal some information about the validity of the model. The results consistently indicate that the beads were placed less than a centimeter away from the position of first measurement, sometimes as little as three millimeters away, and that the beads took between one and two seconds to enter the camera's viewing range. These distance measurements represent a considerable underestimate of actual experimental results.

When the model was modified to force beads to begin at least a centimeter away from the initial measurement position, it returned distances that ranged from 1.5 to 2 centimeters, and found that the beads took 2-10 seconds to enter the camera's viewing range, which is closer to what occurred in the experimental setup. The restriction of initial conditions caused the values of γ to change as well, becoming nearly 2.5 in the case of the 0.63 mm wire and 3.0 for the 0.37 mm wire.

The authors have been unable to find a reason for the change in γ as the wire size changes. It was thought that the change might cause a difference in rates of acceleration for bead on the different wires, but the rigid body model of the friction force causes acceleration to happen nearly instantaneously for $\gamma > 0.1$, see Figure (4). For each $\gamma \geq 0.01$, the bead had already reached terminal velocity by the time the first measurement had occurred—all measurements were taken at least one second after the bead was released. This fact casts doubt on acceleration-based explanations for the change in γ between wire sizes.

This mechanical setup, with its reliance on experimental data to estimate the frictional force, is unable to produce any information about a possible relationship between volume and velocity. Deriving a frictional force that is independent from experimental data may resolve this issue. However, explaining the interaction between the honey and the wire by using a frictional force is not necessarily appropriate. The model presented above does not account for the contact surface area or the energy dissipation due to viscous forces inside the bead. Also, significant changes to the value of γ resulted in a negligible change to the velocity profile over time of a bead. A different approach that captures more of the subtleties of the problem is necessary; however, that may compromise the simplicity of the formulation.

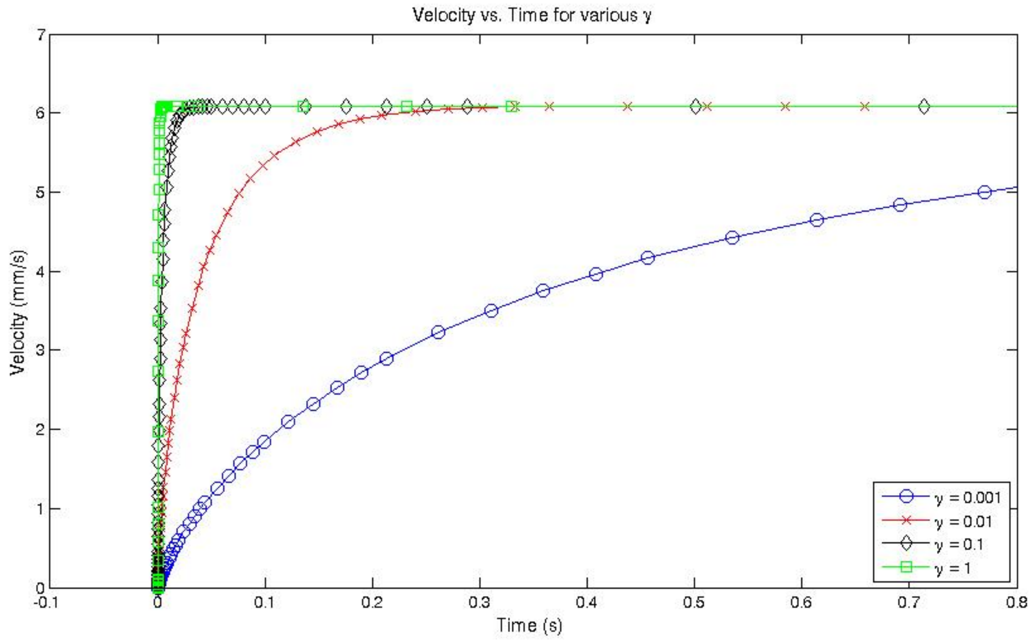


Figure 4: Velocity profiles for a 0.37 mm wire under differing γ.

4 Conclusions

Experimental observations of beads of honey sliding down a vertical wire demonstrated that the velocity of the bead depends roughly linearly on the size of the bead. Statistical analysis of the results demonstrated that the velocity and volume of a bead did not change significantly over the course of a single experiment, though the relationship between mass and velocity was found to be statistically significant.

A simple rigid body model was formulated in an attempt to describe the motion of an isolated bead over time, but was found to be inadequate due to the number of parameters that required fitting with the experimental data. Within the model, it was assumed that the frictional force took the form of a power law dependent on the velocity, $F \propto \dot{x}^{\gamma}$. Numerical results show that this model predicts nearly instantaneous acceleration for reasonable values of γ, i.e., $\gamma \geq 0.1$. For these reasons, the authors believe that the current model cannot properly account for the motion of an isolated bead on a vertical wire.

5 Potential Avenues for Further Research

The rich dynamics present in this system suggest a wide variety of potential avenues of investigation. One such possibility is to compare the motion of a bead on a clean wire to the motion of a bead on a wire with a preexisting coating of fluid. Based on the available literature regarding liquid-surface interactions, the dynamics governing these two systems should be considerably different [5].

However, it is expected that both systems should display an increase in bead velocity with an increase in bead size. An interesting problem is attempting to quantify the difference in how velocity changes as a function of mass, as a means of shedding light on the extent of the influence of preexisting thin films on the behavior of beads.

Another potential direction is to examine more closely the loss of mass from the bead as it moves down the wire due to the thin film trail it leaves behind. The experimental results indicate that a loss of mass causes a decrease in bead velocity; however, it was observed that the individual beads had relatively constant velocities. Whether this can be attributed to the fact that the portion left behind is quite small relative to the size of the bead, or if there is another more subtle mechanism at work, remains unclear. Furthermore, determining a precise estimate for the amount of mass left in the trail, and how this is affected by the changing mass of the parent bead, is yet another open problem. Some insight into this area was obtained through observing the trail of beads that developed out of the deposited film due to the Plateau-Rayleigh instability [2]. It was observed that each bead left behind a regular trail of beads that appeared to decrease in size down the wire, indicating that the mass of the trail decreases with decreasing bead size.

A further aspect of the dynamics of this system, which has been ignored, concerns the activity of the fluid contained within the bead itself, and how this activity reflects upon the observable motion of the entire bead. Inadvertently, some of this activity was observed in an experimental trial, when air bubbles were accidentally injected into the bead. These bubbles were observed to move in a circular, rolling motion within the bead as it moved down the wire, indicating that the fluid within the bead was circulating in a similar fashion. This observation also suggests that the surface of the bead is. in effect, "rolling" down and onto the surface of the wire, indicating that the honey is being deposited onto the wire in a way reminiscent of the "tank tread" analogy described in the literature [5].

6 Acknowledgments

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