Math 263–001 Test 3
Instructor: David Love April 13, 2012

- Show **all** algebraic work to receive full credit.
- Please turn OFF all cell phones, pagers, and other communication devices and put them out of sight.
- All textbooks, notes, etc. must be put away. One 3×5 note card is allowed.
- A t-table is on the back page. Remove it if you would like.

Student's Name (please print): Key

1. We have discussed two statistics, t and z. When is each used?

z: When stdev is known, or when proportions are used

t: When stdev is unknown, and the sample stdev is used to estimate it.

2. Suppose the random variable X has distribution $B(11, 0.6)$. Find $P(X > 6)$.

$P(X>6) = 1 - P(X>6)^c = 1 - P(X \leq 6)$

$= 1 - \text{binomcdf}(11, 0.6, 6)$

$= 0.53277$

3. As the confidence level C increases, the length of the confidence interval decreases. T/F? Explain.

F. To have higher confidence you need a larger interval.

4. Suppose that 60% of the student body favors the construction of a new building on campus.

   (a) What are the mean and standard deviation of the proportion of students in an SRS of size 200 that favors the construction of a new building on campus?

$$\mu_{\hat{p}} = p = 0.6$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.6(0.4)}{200}} = 0.0346$$

   (b) What is the probability that the sample proportion $\hat{p}$ of students in an SRS of size 200 is greater than 50%?

Normal approx:

$$P(\hat{p} > 0.5) = P\left(Z \geq \frac{0.5 - \mu_{\hat{p}}}{\sigma_{\hat{p}}}\right)$$

$$= \text{normalcdf}(-2.8868, E99) = 0.998$$

With binomial distr:

$$P(X > 100) = 1 - P(X \leq 100)$$

$$= 1 - \text{binomcdf}(200, 0.6, 100)$$

$$= 0.997$$

   (c) How large a sample is required to reduce the standard deviation of the proportion of students that favors the construction of a new building on campus to 1%?

Want $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \leq 0.01$

$$\longrightarrow n \geq \frac{p(1-p)}{0.01^2} = \frac{0.6(0.4)}{0.01^2} = 2400$$

5. Suppose $H_0 : \mu = \mu_0$. Set up a one-sided H_a.

$H_a : \mu > \mu_0$

or

$H_a : \mu < \mu_0$

6. Errors in measurements often have a normal distribution. The measured amount of energy released in a certain chemical reaction is normally distributed with $\mu = 12J$ and $\sigma = 2J$. What is the distribution of the average of 20 measurements?

CLT. The sample average has dist. $N(12, \frac{2}{\sqrt{20}}) = N(12, \frac{1}{\sqrt{5}})$

$= N(12, 0.44721)$

7. For every statistical procedure we have studied, there is a single fundamental requirement for the sample on which the procedure is based. What is it?

The samples must be SRS, or independent.

8. A p-value of 0.05 is stronger evidence against the null hypothesis than a p-value of 0.1. T/F? Explain.

True.

Smaller p-value means less probability of wrongly rejecting H_0.

9. Experiments on learning in animals sometimes measure how long it takes mice to find their way through a maze. The mean time is 18 seconds for one particular maze. A researcher thinks that a loud noise will cause the mice to complete the maze in less time. She measures how long each of 10 mice takes with a noise as stimulus. What are the null and alternative hypotheses for this test?

$H_0: \mu = 18s$

$H_a: \mu < 18s$

10. A critical dimension of an automobile crankshaft is supposed to be 224 mm, and the variability is unknown. A sample of 30 has $\bar{x} = 224.002$, and $s = 0.0618$. Is there evidence that the population mean for this dimension is not 224 mm?

(a) State the hypotheses

$H_0: \mu = 224$ mm

$H_a: \mu \neq 224$ mm.

(b) Find the test statistic.

$$t = \frac{224.002 - 224}{0.0618/\sqrt{30}} = 0.1773$$

(c) Find the p-value by calculator or by table. What do you conclude?

Calculator: p-value $= 2\,\text{tcdf}(0.01773, \text{E}99, 29)$

$= 0.8605$

Table: The smallest critical value in the table is 0.683, which would have a p-value of $2(0.25) = 0.5$. Thus the p-value must be larger than 0.5.

Do not reject H_0, shaft mean is 224 mm.

11. Assume two independent samples yield $\overline{x}_1 = 48, \overline{x}_2 = 53, s_1 = 7.2, s_2 = 4, n_1 = 61$ and $n_2 = 41$. Find a 95% CI for the difference in the corresponding values of μ. Does this interval contain more or fewer values than a 99% CI?

$$CI = (\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \qquad t^* = invT\left(\frac{1.95}{2}, 40\right) = 2.02$$

Then $CI = (-7.25, -2.75)$

Fewer values than a 99% CI.

12. The English mathematician John Kerrich tossed a coin 10,000 times and recorded 5067 heads.

(a) Is this significant evidence at the 5% level that the probability that Kerrick's coin comes up heads is not 0.5?

$H_0: p = 0.5$

$H_a: p \neq 0.5$

$$Z = \frac{0.5067 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{10\,000}}} = 1.34, \quad p\text{-value} = 2\,normalcdf(1.34, E99) = 0.18$$

$\hat{p} = \frac{5067}{10\,000} = 0.5067$

No evidence that probability is not 0.5.

(b) Find a 95% confidence interval for the proportion of heads in this experiment.

$$CI = 0.5067 \pm z^* \sqrt{\frac{0.5067(1-0.5067)}{10,000}}$$

$= invNorm\left(\frac{1.95}{2}\right) = 1.96$

$= (0.4969, 0.5165)$

13. Two computerized timers measure the time for a toy car to complete laps around a track. For each lap, the measurements are

Lap	1	2	3	4
Timer 1	14	18	17	15
Timer 2	12	18	11	13
Difference	2	0	6	2

The manufacturers want to test whether the timers are producing different results.

(a) Find the sample mean and standard deviation of the difference in the means.

Mean of differences = 2.5

stdev " " = 2.5166

(b) State the hypotheses.

$H_0: \mu = 0$

$H_a: \mu \neq 0$

(c) Find the test statistic. Is this a z test or a t test?

$$t = \frac{2.5 - 0}{2.5166/\sqrt{4}} = 1.986$$