Math 112–008
Instructor: David Love

Key

Mini-Test
May 1, 2009

- No partial credit will be given for multiple choice problems.
- For all others, show **all** algebraic work to receive full credit.
- Please turn OFF all cell phones, pagers, and other communication devices and put them out of sight.
- All textbooks, notes, etc. must be put away.

Student's Name (please print): ______________________________

By signing my name below, I agree that I am following all rules and regulations set forth by the Code of Academic Integrity. Furthermore, I agree that I am following all rules set by my instructor and by the course policy for this exam. This includes ensuring that all calculator programs except possibly EVALUATE and QUADRATIC FORMULA have been deleted.

Student's Signature: ______________________________ Date: ____________________

Note that the final is on May 11th, 8:00-10:00 am, CESL 102.

Some formulas that may be useful:

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} \qquad\qquad A(t) = Pe^{rt}$$

1. Rewrite in exponential form: $\log_b(M) = E$. Choose the best answer.

a) $b^E = M$ b) $b^M = E$ c) $M^b = E$ d) $E^b = M$ e) None of these

2. Determine whether each of the following demonstrates correct use of the properties of logarithms and/or exponents. If it is correct, circle "Correct," if not circle "Incorrect" and correct it (if it can be corrected).

(i) $\log_b(t-r) = \dfrac{\log_b(t)}{\log_b(r)}$

$\log_b\left(\frac{t}{r}\right) = \log_b(t) - \log_b(r)$

Correct Incorrect

(ii) $\log_b(2b^3) = \log_b(2) + 3$

$\log_b(2b^3) = \log_b(2) + \log_b(b^3)$

$= \log_b(2) + 3\log_b(b)$

$= \log_b(2) + 3$

Correct Incorrect

(iii) $\log_4\left(\dfrac{16x}{y^2}\right) = 2 + \log_4(x) - 2\log_4(y)$

$\log_4\left(\frac{16x}{y^2}\right) = \log_4(16x) - \log_4(y^2)$

$= \log_4(16) + \log_4(x) - 2\log(y)$

$= 2 + \log_4(x) - 2\log_4(y)$

Correct Incorrect

(iv) $\ln(4+2) = \ln(4) + \ln(2)$

$\ln(4) + \ln(2) = \ln(4 \cdot 2)$

$= \ln(8)$

Correct Incorrect

3. Use your calculator to approximate the value of $\log_7(18)$ to the nearest 0.0001. Show what you entered into your calculator.

$$\frac{\ln(18)}{\ln(7)} = \frac{\log(18)}{\log(7)} = 1.4854$$

4. Write the following expression as a single logarithm, $\ln(P)$, where P is a simplified expression. Choose the best answer

$$\ln(x^2-1) - \ln(x-1) - \ln(x+1) + \underbrace{\cancel{\ln(1)}}_{0}$$

$$= \ln\left(\frac{x^2-1}{x-1}\right) - \ln(x+1)$$
$$= \ln\left(\frac{x^2-1}{(x-1)(x+1)}\right) = \ln\left(\frac{x^2-1}{x^2-1}\right) = \ln(1)$$
$$= 0$$

a) e

b) 1

c) 0

d) $\ln\left(\frac{1}{x-1}\right)$

e) $\ln(x+1)$

5. Expand the following logartihmic expression completely.

$$\log\left(5x^7\sqrt[3]{y}\right)$$

$$\log(5) + \log(x^7) + \log(\sqrt[3]{y})$$
$$= \log(5) + 7\log(x) + \frac{1}{3}\log(y)$$

6. Solve each equation for the variable (give exact answers only)

(i) $7^{3-4x} = 12$ $\rightarrow (3-4x)\log 7 = \log 12$

$\rightarrow 3-4x = \dfrac{\log 12}{\log 7} \rightarrow 3 - \dfrac{\log 12}{\log 7} = 4x$

$\rightarrow \boxed{x = \dfrac{1}{4}\left[3 - \dfrac{\log 12}{\log 7}\right]}$

(ii) $4^x = 9^{2x-1}$

$\rightarrow x\log 4 = (2x-1)\log 9 \rightarrow x\log 4 = 2x\log 9 - \log 9$

$\rightarrow x\log 4 - 2x\log 9 = -\log 9$

$x(\log 4 - 2\log 9) = -\log 9 \rightarrow \boxed{x = \dfrac{-\log 9}{\log 4 - 2\log 9}}$

(iii) $\log_{11}(x+97) = 3$

$\rightarrow 11^3 = x+97 \rightarrow x = 11^3 - 97 = \boxed{1234 = x}$

(iv) $\log\left(\frac{1}{3x}\right) + \log(x^2) = \log(1)$

$\log\left(\dfrac{1}{3\cancel{x}} \cdot x^{\cancel{2}}\right) = \log(1)$

$\rightarrow \log\left(\dfrac{1}{3}x\right) = \log(1)$

$\rightarrow \dfrac{1}{3}x = 1$

$\rightarrow \boxed{x = 3}$

6. Solve each equation for the variable (give exact answers only)

(i) $7^{3-4x} = 12$ $\to$ $3-4x = \log_7(12)$

$\to$ $-4x = \log_7(12) - 3$

$\to$ $\boxed{x = \dfrac{\log_7(12)-3}{-4} = \dfrac{3-\log_7(12)}{4}}$

(ii) $4^x = 9^{2x-1}$

$\to$ $x = (2x-1)\log_4(9) = 2x\log_4(9) - \log_4(9)$

$\to$ $x + \log_4(9) = 2x\log_4(9)$

$\to$ $\log_4(9) = 2x\log_4(9) - x$

$= x(2\log_4(9) - 1)$ $\to$ $\boxed{x = \dfrac{\log_4(9)}{2\log_4(9)-1}}$

(iii) $\log_{11}(x+97) = 3$

(iv) $\log\left(\frac{1}{3x}\right) + \log(x^2) = \log(1)$

6. Solve each equation for the variable (give exact answers only)

(i) $7^{3-4x} = 12$

(ii) $4^x = 9^{2x-1}$

$\rightarrow x \log_9(4) = 2x - 1$

$\rightarrow 1 = 2x - x \log_9(4)$

$\rightarrow 1 = x(2 - \log_9(4))$

$\rightarrow \boxed{x = \dfrac{1}{2 - \log_9(4)}}$

(iii) $\log_{11}(x + 97) = 3$

(iv) $\log\left(\frac{1}{3x}\right) + \log(x^2) = \log(1)$

7. A radioactive substance decays so that 80% of the original amount remains after 9 years. Use an exponential model to determine the half-life of the substance to the nearest year.

$$.8P = P\left(\tfrac{1}{2}\right)^{9/h} \rightarrow .8 = \left(\tfrac{1}{2}\right)^{9/h}$$

$$\rightarrow \log(0.8) = \frac{9}{h}\log\left(\tfrac{1}{2}\right) \rightarrow h\log(0.8) = 9\log\left(\tfrac{1}{2}\right)$$

$$\rightarrow \boxed{h = \frac{9\log\left(\frac{1}{2}\right)}{\log(0.8)} \approx 28 \text{ years}}$$

8. You invest \$3,000 at a rate of 5%, compounded continuously. After how many years (to the nearest 0.01) will your investment have grown to \$7,000?

$$\$7000 = \$3000\, e^{0.05t} \rightarrow \frac{7}{3} = e^{0.05t}$$

$$\rightarrow 0.05t = \ln\left(\tfrac{7}{3}\right)$$

$$\rightarrow \boxed{t = \frac{\ln\left(\frac{7}{3}\right)}{0.05} \approx 16.95 \text{ years}}$$