Math 112–008 Midterm #3
Instructor: David Love Name: Key April 15, 2009

- No partial credit will be given for multiple choice problems.
- For all others, show **all** algebraic work to receive full credit.
- Please turn OFF all cell phones, pagers, and other communication devices and put them out of sight.
- All textbooks, notes, etc. must be put away.

Student's Name (please print): _____
By signing my name below, I agree that I am following all rules and regulations set forth by the Code of Academic Integrity. Furthermore, I agree that I am following all rules set by my instructor and by the course policy for this exam. This includes ensuring that all calculator programs except possibly EVALUATE and QUADRATIC FORMULA have been deleted.

Student's Signature: _____ Date: _____

Some formulas that may be useful:

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} \qquad A(t) = Pe^{rt}$$

1. Describe the end behavior of each function listed below. Choose the best option.

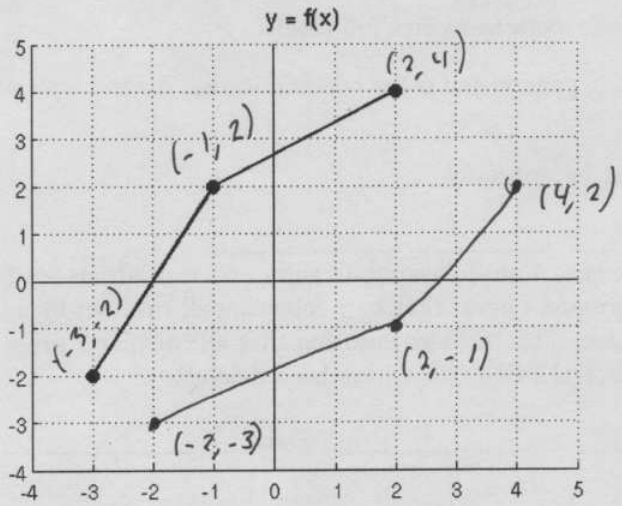
(i) $y = -x^4 + 12x^3$

c) $y \to \infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to \infty$
e) $y \to 0$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$
d) $y \to -\infty$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$
a) $y \to \infty$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$
b) $y \to -\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to \infty$

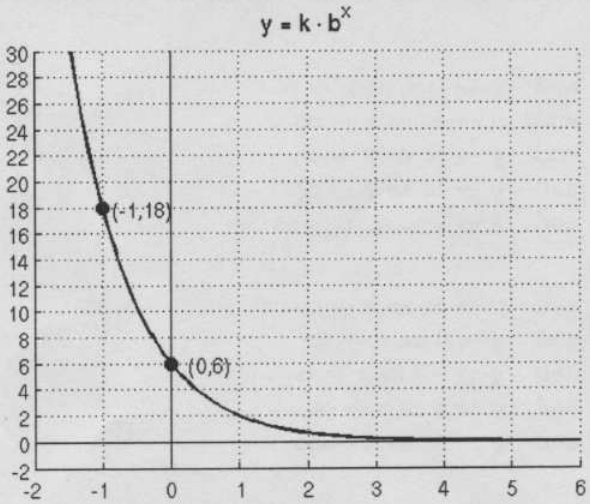
(ii) $y = e^{-x} + 5$

c) $y \to 5$ as $x- \to -\infty$ and $y \to \infty$ as $x \to \infty$
b) $y \to \infty$ as $x- \to -\infty$ and $y \to 0$ as $x \to \infty$
e) $y \to \infty$ as $x- \to -\infty$ and $y \to -5$ as $x \to \infty$
d) $y \to \infty$ as $x- \to -\infty$ and $y \to 5$ as $x \to \infty$
a) $y \to 0$ as $x- \to -\infty$ and $y \to \infty$ as $x \to \infty$

2. Below is a plot of $y = f(x)$. Sketch, on the same axes, the graph of $y = f^{-1}(x)$.



3. The graph of $y = k \cdot b^x$ is shown below. Determine the values of k and b.



Plug in (0, 6)

$6 = k \cdot b^0$

$6 = k \cdot 1$

$k = 6$

Plug in (-1, 18)

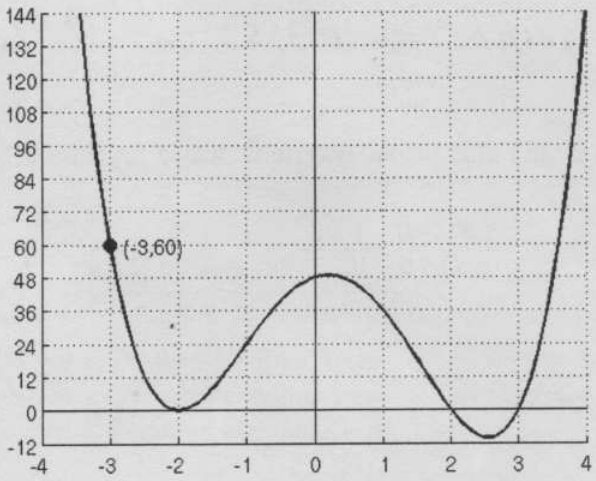
$18 = 6b^{-1} = \frac{6}{b}$

$\frac{18b}{18} = \frac{6}{18}$

$\rightarrow b = \frac{1}{3}$

$y = 6\left(\frac{1}{3}\right)^x$

4. Write an equation for the polynomial function graphed below. Leave your answer in factored form.



$y = a(x+2)^2(x-2)(x-3)$

$60 = a(-1)^2(-5)(-6)$

$60 = 30a$

$\rightarrow a = 2$

$y = 2(x+2)^2(x-2)(x-3)$

5. At what interest rate (to the nearest 0.01%) compounded twice a year must \$3000 be invested for 1 year so that the accumulated amount is \$3180?

$$\frac{3180}{3000} = \frac{3000\left(1+\frac{r}{2}\right)^{2\cdot 1}}{3000}$$

$$1.06 = \left(1+\frac{r}{2}\right)^2 \qquad \sqrt{\ }$$

$$\sqrt{1.06} = 1+\frac{r}{2}$$

$$1.02956 = 1+\frac{r}{2} \Rightarrow 5.91\%$$

6. If $f(x)$ is a one-to-one function and $f(5) = -3$, what point MUST be on the graph of $f^{-1}(x)$?

a) $(5, -3)$ b) $(\frac{1}{5}, -3)$ c) $(\frac{1}{5}, \frac{-1}{3})$ d) $(-3, 5)$ e) $(3, 5)$

$(5, -3)$ on original graph $\Rightarrow$ $(-3, 5)$ on $f^{-1}(x)$

7. Given the tables for functions $f(x)$ and $g(x)$ below, determine which function is one-to-one. Justify your answer.

x	0	1	2
$f(x)$	-1	4	-1

Same, so not 1-to-1

x	0	1	2
$g(x)$	2	0	1

$\leftarrow$ Is 1-to-1.

8. Given the function $h(x)$ from the table below has an inverse, construct the inverse function $h^{-1}(x)$ in the space provided.

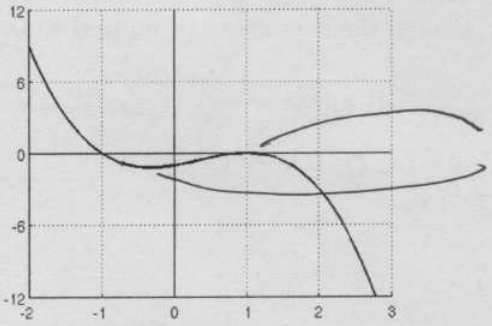
x	0	2	5	6
$h(x)$	-1	3	2	4

x	-1	2	3	4
$h^{-1}(x)$	0	5	2	6

9. Write out the terms of the sum and simplify.

$$\sum_{n=2}^{4} 3n = 3(2) + 3(3) + 3(4)$$
$$= 6 + 9 + 12 = 27$$

10. From the complete graph of a polynomial below, determine the minimum degree and sign of the leading term. Choose the best answer.



Two turning points
⇒ degree 3

a) Degree 2, Positive Leading Term
b) Degree 3, Positive Leading Term
c) Degree 3, Negative Leading Term
d) Degree 4, Positive Leading Term
e) Degree 4, Negative Leading Term

11. Lee invests \$4,000 in a bank account which pays 4% annual interest.

(i) If the interest is compounded **quarterly**, how much money will be in the account after 3 years?

a) \$4499.46 b) \$4507.30 c) \$4509.99 d) \$4551.52 e) \$4572.32

$$4000\left(1+\frac{0.04}{4}\right)^{4\cdot 3}$$

(ii) If the interest is compounded **continuously**, how much money will be in the account after 3 years?

a) \$4499.46 b) \$4507.30 c) \$4509.99 d) \$4551.52 e) \$4572.32

$$4000\, e^{0.04\cdot 3}$$

12. A cup of coffee at an initial temperature of 149°F is set in a room where the constant temperature is 72°F. The rate of change r is -0.045 when t is measured in minutes, and the temperature after t minutes is modeled by: $A(t) = C + (A_0 - C)e^{rt}$. Find the temperature of the coffee after half an hour has passed. Choose the best answer (rounded to the nearest °F).

a) 89°F (b) 92°F c) 113°F d) 130°F e) 147°F

$A(t) = 72 + (149 - 72)e^{-0.045t}$

$t = 30$ min

13. $f(x) = 3x - 2$ and $g(x) = \dfrac{x+2}{3}$

(i) Verify algebraically that $f(x)$ and $g(x)$ are inverse functions.

$f(g(x)) = 3\left(\frac{x+2}{3}\right) - 2 = x + 2 - 2 = x$

$g(f(x)) = \dfrac{(3x-2)+3}{3} = \dfrac{3x}{3} = x$

(ii) Find $f(g(-3))$.

By above, we know that $f(g(x)) = x$.

So $\boxed{f(g(-3)) = -3}$

14. Find the inverse of the one-to-one function $F(x) = \dfrac{5}{10+x}$. Show all steps.

$y = \dfrac{5}{10+x}$

Inverse:

$(10+y)x = \dfrac{5}{10+y}(10+y)$

$\dfrac{x(10+y)}{x} = \dfrac{5}{x}$

$10 + y = \dfrac{5}{x}$

$-10 \qquad -10$

$\boxed{y = \dfrac{5}{x} - 10}$

15. Find all real zeros of $P(x) = x^3 - 3x^2 - 5x + 15$, given that $x = 3$ is one of the zeros. Answers must be exact, and all division and other work must be shown for credit.

$$\begin{array}{r|rrr} 3 & 1 & -3 & -5 & 15 \\ & & 3 & 0 & -15 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$x^2 - 5 = 0$

$\Rightarrow x = \pm\sqrt{5}$

$\Rightarrow \boxed{x = 3, \pm\sqrt{5}}$

16. Divide $P(x)$ by $d(x)$

$$P(x) = x^3 - 6x^2 + 9x - 7 \qquad d(x) = x^2 - 2x + 5$$

The remainder is (choose the best answer):

a) $2x - 1$ b) $-4x + 13$ c) $5x + 6$ d) $3x^2$ e) 18

$$\begin{array}{r} x - 4 \\ x^2 - 2x + 5 \overline{)\, x^3 - 6x^2 + 9x - 7} \\ \underline{x^3 - 2x^2 + 5x} \\ -4x^2 + 4x - 7 \\ \underline{-4x^2 + 8x - 20} \\ -4x + 13 \end{array}$$