Math 112–008 | Midterm #1
Instructor: David Love   Name: February 6, 2009

- No partial credit will be given for multiple choice problems.
- For all others, show **all** algebraic work to receive full credit.
- Please turn OFF all cell phones, pagers, and other communication devices and put them out of sight.
- All textbooks, notes, etc. must be put away.

Student's Name (please print): ________________________________
By signing my name below, I agree that I am following all rules and regulations set forth by the Code of Academic Integrity. Furthermore, I agree that I am following all rules set by my instructor and by the course policy for this exam. This includes ensuring that all calculator programs except possibly EVALUATE and QUADRATIC FORMULA have been deleted.

Student's Signature: ________________________________ Date: ____________________

1. Let $g(t) = \frac{3t+2}{t-1}$. Find $g(t-2)$ and simplify as much as possible.

   Solution:

$$\begin{aligned} g(t-2) &= \frac{3(t-2)+2}{(t-2)-1} \\ &= \frac{3t-6+2}{t-2-1} \\ &= \frac{3t-4}{t-3} \end{aligned}$$

   Several people made the mistake of writting $(t-2)-1 = -t+2$, i.e., multiplying by -1 rather than just subtracting 1 from $t-2$. Note, if $t-1 \neq t(-1)$, then $(t-2)-1 \neq (t-2)(-1)$. □

2. Find $r(-1)$ given that

$$r(x) = \begin{cases} 2x+3, & x \leq 1 \\ x^2-4, & x > 1 \end{cases}$$

   a) 3   b) -1 c) 5   d) 1   e) -3

   Solution: We can see that the input, -1, fits into the area $x \leq 1$, since $-1 \leq 1$. Then just evaluate $r(-1) = 2(-1)+3 = -2+3 = 1$. The answer is d. □

3. Find the domain of the function $h(x) = \dfrac{3x+2}{2x^2+6x}$. Show your work and express your answer in interval notation.

Solution: To find the domain of a fraction, we must avoid the ares where the denominator is zero (why?). Then we need

$$2x^2 + 6x \neq 0$$
$$2x(x+3) \neq 0$$

which we can split into two problems given by $2x \neq 0$ and $(x+3) \neq 0$. Solving these gives $x \neq 0$ and $x \neq -3$. However, x can be any other point.

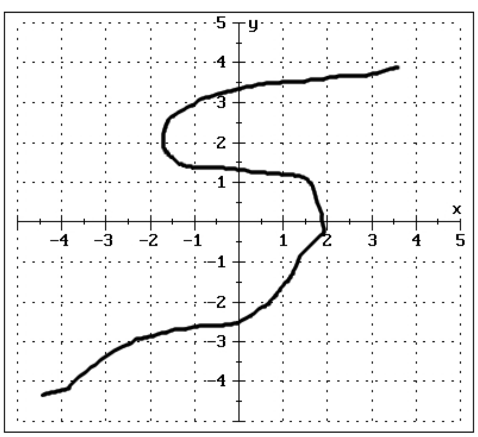
To find the interval notation expression, notice that x can be smaller than -3, between -3 and 0, or larger than 0. To say this in interval notation, write $(-\infty, -3) \cup (-3, 0) \cup (0, +\infty)$. □

4. Find the domain of the function $\sqrt{4-2x}$.

a) $[1, +\infty)$ b) $(3, +\infty)$ c) 2 d) $(-\infty, -3)$ e) $(-\infty, 2]$

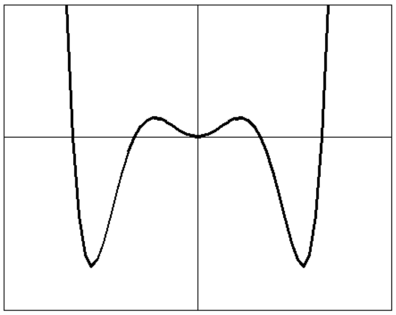
Solution: This domain problem has a square root, so the "rule" is that the domain is the values of x for which the inside of the square root is positive (why?). Then we need to find $4 - 2x \geq 0$. Adding $2x$ to both sides gives $4 \geq 2x$, and dividing by 2 gives $2 \geq x$, or $x \leq 2$. This is the same as $-\infty < x \leq 2$, or in interval notation is $(-\infty, 2]$. The answer is e. □

5. Sketch a graph that does **NOT** represent y as a function of x. Briefly explain why it is not a function.



Solution: This graph does not pass the vertical line test. That means that certain values of the input have more than one output. □

6. The graph below is a graph of the function $f(x)$. Use this to determine if $f(x)$ is even, odd or neither. Explain your answer.

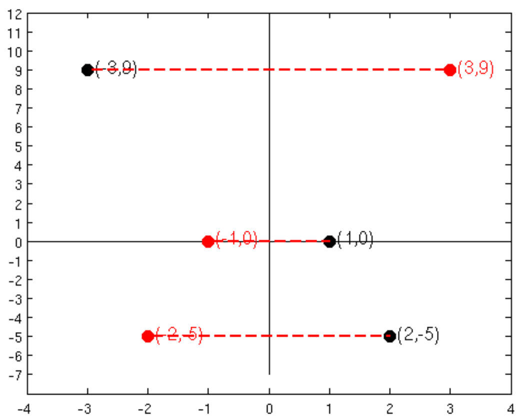


Solution: The graph of f is symmetric across the $y-$axis. Thus the graph is even. □

7. The table below contains a few points from a function $g(x)$. You are told that $g(x)$ is an even function. Using the fact that $g(x)$ is even, fill in the rest of the values of the table.

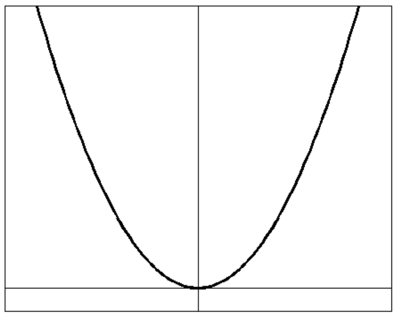
x	-3	-2	-1	1	2	3
$g(x)$	9			0	-5	

Solution: Plot each point on a graph, then reflect them over the $y-$axis, as in the figure below

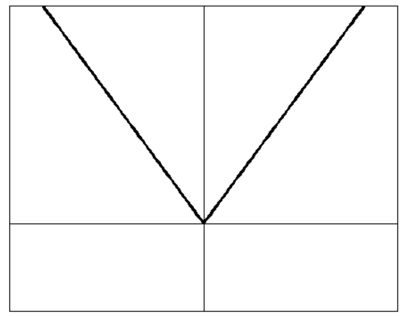


□

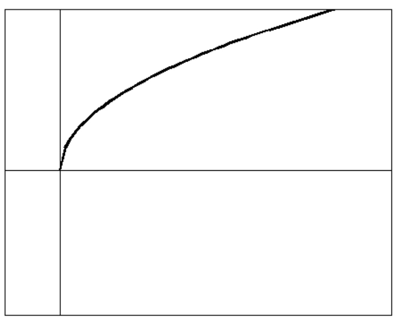
8. Identify each graph below as one of the following types of functions: Constant, Identity, Squaring (Quadratic), Cubing (Cubic), Square Root, Absolute Value, Reciprocal, Semicircle.



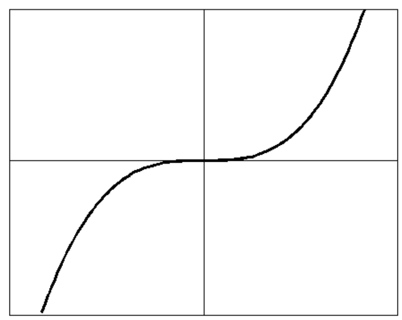
Function: Squaring



Function: Absolute Value

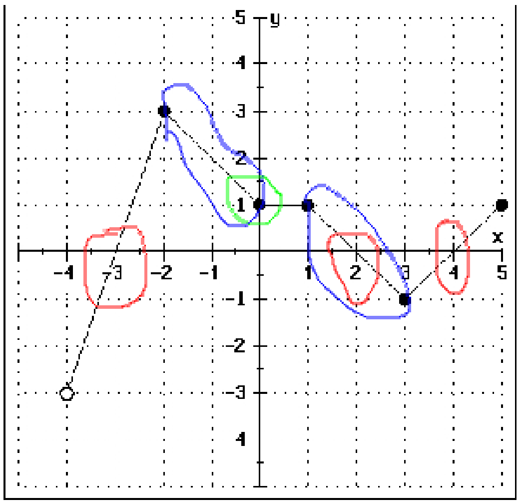


Function: Square Root



Function: Cubing

9. Use the graph of $y = f(x)$ below to answer the following questions (express answers in interval notation where appropriate).



(a) What is the domain of f? Choose the best answer:

a) $(-4, 5]$ b) $[-4, 5]$ c) $[-3, 3]$ d) $(-3, 3]$ e) None of these

Solution: To find the domain, look at the x values that have a corresponding y value. The open circle at the left end of the graph means that the function takes no value at $x = -4$. The function exists at every point after that until $x = 5$, inclusive (because of the closed circle). Then the domain is $-4 < x \leq 5$, or $(-4, 5]$. The answer is a. □

(b) Determine the open intervals over which f is decreasing.

Solution: f is decreasing over the areas where the graph goes down as it moves to the right, and is circled in blue on the graph. This happens between $x = -2$ and $x = 0$, and again between $x = 1$ and $x = 3$. These intervals are $(-2, 0)$ and $(1, 3)$. The graph is **not** decreasing on $(0, 1)$. It is constant there. □

(c) Find the zeros of f.

Solution: The zeros are where the graph crosses the $x-$axis, and are circled in red on the graph. This happens at $x = -3$, $x = 2$ and $x = 4$. □

(d) Find the y-intercept of f.

Solution: The $y-$intercept is where the graph crosses the $y-$axis, or when $x = 0$. This is circled in green on the graph, and happens at $y = 1$. □

10. The length of a rectangle is 4 times as long as the width. Express the perimeter of the rectangle as a function of the width. Simplify your answer.

    Solution: We want to know the perimeter, and we know that perimeter is given by $P = 2w+2l$. We are told that the length is four times as long as the width, which mathematically says that $l = 4w$. To determine what the paremeter is with this additional information, we substitute $l = 4w$ into the perimeter equation to get

$$P = 2w + 2(4w)$$
$$P = 2w + 8w$$
$$P = 10w$$

□

11. A ball is thrown into the air from ground level, and its height (in feet) t seconds after the throw is given by the function $h(t) = -16t^2 + 208t$.

    (a) Use algebra to find the zero(s) of h.

    Solution: Find the zeros by

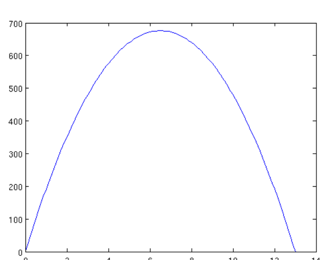
$$-16t^2 + 208t = 0$$
$$t(-16t + 208) = 0$$

    and split into two problem with $t = 0$ and $-16t + 208 = 0$. The first is already solved, so we subtract 208 from both sides in the second to get $-16t = -208$, and divide by -16 to get $t = 13$. Then the zeros are $t = 0$ and $t = 13$. □

    (b) Give a physical explanation of what these zeros mean (in terms of what's happening to the ball there).

    Solution: The zeros are when the ball is on the ground, either before it is thrown or when it lands. □

    (c) Use your calculator to sketch a graph of what the path of the ball looks like as it travels through the air. **Your graph should include all zeros of h(t).**

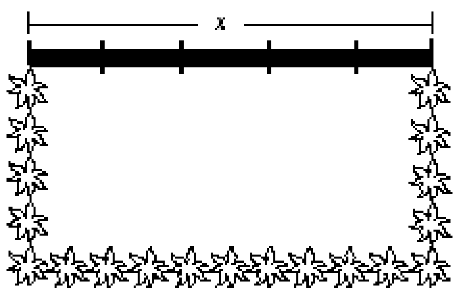


12. A manufacturer of televisions find that the cost (in dollars) generated by manufacturing x units per week is given by the function $C(x) = -12x^2 + 15x + 3000$.

    Find the y-intercept of the function, and explain what it represents in terms of dollars and number of units manufactured.

    Solution: As in one previous problem, the $y-$intercept happens when the graph intersects the $y-$axis, i.e., when $x = 0$. Then just plug in $x = 0$ to get $C(0) = -12(0)^2+15(0)+3000 = 3000$. This is the cost of manufacturing no televisions at all. □

13. A rectangular area of 4000 square feet is to be enclosed by shrubs on three sides and fence on the fourth side as shown in the diagram below. The shrubs cost \$20 per foot and the fence costs \$15 per foot. Express the total cost of the project as a function of x, the length of the fence. Simplify your answer



Solution: We know that the area of a rectangle is $A = wl$. In the picture, the length is labeled x, so $A = wx$. We are told that the area is 4000 square feet, so $4000 = wx$. We will want our final formula as a function of x and not w, so solving the area equation for w gives $w = 4000/x$.

This problem deals with the cost of the enclosure, so we need an expression for the cost. The enclosure will have x feet of fence, which costs \$15 per foot, so the total cost of the fence is $15x$.

The other three sides of the garden are made up of shrubs, which cost \$20 per foot. Both of the widths are made up of shrubs, and so is one of the lengths, giving a total of $2w + x$, so the cost is $20(2w + x)$. Then the total cost of the project is $15x + 20(x + 2w) = 35x + 40w$.

The only thing left to do is to substitute in for w from the equation for area, which gives

$$C(x) = 35x + 40\frac{4000}{x} = 35x + \frac{160,000}{x}$$

□