Problem 1 Express the area of a square as a function of its perimeter p.

Solution: In class we saw several examples application problems involving rectangles. Given this context, you should ask yourself: what is the difference between a square and a rectangle? The answer is that all four sides of a square are the same length. This may seem like a silly question, but at least half of the homeworks did not mention this fact at all. Those who did write this down used the letter s to represent the side of a square, so I will do the same.

Let's catalog some of the things we know about this problem:

1. We know that the area of a square is $A = s^2$ (this comes from the rectangle formula, where $A = lw$, but we know that the length and width are the same, or $l = s$ and $w = s$).
2. We can find the formula for the perimeter of a square by noticing that (1) a square has four sides and (2) each side has length s. Thus we know the perimeter is $P = s + s + s + s = 4s$ (this can also be seen from the rectangle formula, where $P = 2w + 2l$, but width and length are the same, so $l = s$ and $w = s$, then plugging that in gives $P = 2s + 2s = 4s$).
3. Our formula in number 1 gives area in terms of the side. We want area in terms of the perimeter. We should use our formula relating the perimeter to the square to the side length $P = 4s$. We can divide both sides by 4 to get $s = \frac{P}{4}$.

now we plug our new equation for s into the area equation to get

$$A = s^2$$
$$A = \left(\frac{P}{4}\right)^2$$
$$A = \frac{P^2}{4^2}$$
$$A = \frac{P^2}{16}$$

I saw a second mistake come up multiple times on the homework. Some students looked at the solution in the back of the book and saw the 16 in the denominator. They recognized that $16 = 4 \cdot 4$, and assumed that 4 must be the side of the square. This is a a serious misunderstanding of the question. If the square did have side 4, then its perimeter would be $(4 + 4 + 4 + 4 =)16$. Because the perimeter has a specific value, we cannot find the area *as a function of* the perimeter, only the area of a square with perimeter = 16. □

Problem 3 A rectangle has an area of 72 square centimeters. Determine the length as a function of the width w.

Solution: Nearly everyone in the class got this problem correct, although some didn't show any work. We know that the area is given by $A = lw$. We are given that the area is $A = 72\text{cm}^2$. Then we divide both sides of $A = lw$ by the width to get $l(w) = \dfrac{A}{w} = \dfrac{72}{w}$. □

Problem 7 A ladder of length 8 meters leans against a wall. Express the height of the top of the ladder as a function of the distance x from the foot of the wall. What is the domain?

Solution: As we saw in class, the ladder leaning against a wall forms a right triangle. We will say that h is the height of the tip of the ladder. We know the Pythagorean Theorem, so we know $x^2 + h^2 = 8^2$, because the ladder is 8 meters long. Then we isolate h on one side of the equation to get $h^2 = 64 - x^2$. We know that the height will always be a positive number, so we can take the square root and not worry about the negative part of the $\pm$ that normally comes up when you take a square root. Then $h = \sqrt{64 - x^2}$.

To find the domain, we first realize that we will not be dealing with negative values of x, since x is a distance from a wall. Then we must find the area where the part inside the square root is positive or zero. Thus we want $64 - x^2 \geq 0$.

$$\begin{aligned} 64 - x^2 &\geq 0 \text{Subtract 64 from both sides} \\ -x^2 &\geq -64 \text{Multiply by } -1 \\ x^2 &\leq 64 \text{Take the square root} \\ x &\leq 8 \end{aligned}$$

and so the domain is $0 \leq x \leq 8$, which is $[0, 8]$ in interval notation. $\square$