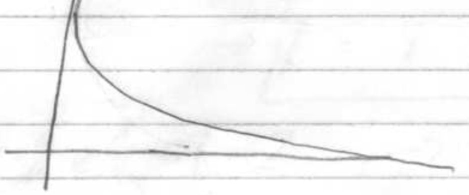
Section 4.5, 2, 9, 13, 16, 20, 21, 27, 30, 34, 35, 38, 41, 44, 47, 49, 50, 54, 55, 57, 58, 60, 63

→ 2) $\log_3 10 = \frac{\log_{10} 10}{\log_{10} 3} = \frac{1}{\log_{10} 3} = \boxed{2.0959}$

9) $y = 5 - \log_2 x = 5 - \frac{\log x}{\log 2}$



13) $4^x = 64$. $64 = 16 \cdot 4 = 4^2 \cdot 4 = 4^{2+1} = 4^3$

$\to 4^x = 4^3 \to \boxed{x = 3}$

16) $\left(\frac{3}{5}\right)^x = \frac{5}{3} \to x \log \frac{3}{5} = \log\left(\frac{5}{3}\right)$

$\to \frac{x(\log 3 - \log 5)}{\log 3 - \log 5} = \log 5 - \log 3 = \frac{-(\log 3 - \log 5)}{\log 3 - \log 5}$

$\to \boxed{x = -1}$

→ 20) $\log_2 x = 6 \to 2^6 = \boxed{x = 64}$

21) $\log_6 x = -2 \to 6^{-2} = \boxed{x = \frac{1}{36}}$

27) $e^{2x-1} = 123 \to (2x-1)\cancel{\ln e} = \ln 123$

$\to 2x - 1 = \ln 123 \to 2x = \ln 123 + 1$

$\to \boxed{x = \frac{\ln 123 + 1}{2} \approx 2.9061}$

30) $\dfrac{140\left(\frac{1}{2}\right)^{t/4}}{140} = \dfrac{350}{140} \rightarrow \left(\frac{1}{2}\right)^{t/4} = 2.5$

$\rightarrow \dfrac{\frac{t}{4}\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)} = \dfrac{\log(2.5)}{\log\left(\frac{1}{2}\right)} \rightarrow \dfrac{t}{4} = \dfrac{\log(2.5)}{\log\left(\frac{1}{2}\right)}$

$\rightarrow \boxed{t = 4\dfrac{\log(2.5)}{\log(0.5)} \approx -5.2877}$

$\rightarrow$ 34) $2e^x - 9 = 2 \rightarrow 2e^x = 11 \rightarrow e^x = \dfrac{11}{2}$

$\rightarrow \boxed{x = \ln\left(\frac{11}{2}\right) \approx 1.7047}$

35) $e^{2x} - 4e^x = 0 \rightarrow e^x(e^x - 4) = 0$

Break up into:

$e^x = 0$ (a)

$e^x - 4 = 0$ (b)

(a) has no solution, since the range of an exponential is $(0, \infty)$.

(b) $e^x - 4 = 0 \rightarrow e^x = 4 \rightarrow \boxed{x = \ln 4 \approx 1.3863}$

38) $\log_7(5x) = 2 \rightarrow 7^2 = 5x \rightarrow \boxed{x = \frac{49}{5} = 9.8}$

41) $\log x + \log(x-15) = 2 \rightarrow \log_{10}(x(x-15)) = 2$

$\rightarrow x(x-15) = 10^2 \rightarrow x^2 - 15x - 100 = 0$

$\rightarrow x = \dfrac{15 \pm \sqrt{15^2 - 4(-100)}}{2} = \dfrac{15 \pm 25}{2} = -5, 20.$

$x = -5$ is not in the domain of $\log x$, so it cannot be a solution.

$\boxed{x = 20}$ is in the domain of both $\log x$ and $\log(x-15)$, so it is the solution.

$\rightarrow$ 44) $3x+5 = \log x - \log\left(\frac{x}{10}\right) = \log\left(\frac{x}{x/10}\right) = \log(10) = 1$

$\rightarrow 3x+5 = 1 \rightarrow 3x = -4 \rightarrow x = \frac{-4}{3}$

But $x = -\frac{4}{3}$ is not in the domain of $\log x$ or $\log\left(\frac{x}{10}\right)$, so there is $\boxed{\text{no solution.}}$

47) $\log(x^2-1) - \log(x+4) = \log x$

$\rightarrow \log\left(\frac{x^2-1}{x+4}\right) = \log x \rightarrow \frac{x^2-1}{x+4} = x$

$\rightarrow x^2 - 1 = x(x+4) = x^2 + 4x$

$-x^2 \qquad\qquad -x^2$

$\rightarrow -1 = 4x \rightarrow x = -\frac{1}{4}$

But $x = \frac{-1}{4}$ is not in the domain of $\log x$ or $\log(x^2-1)$ (it IS in the domain of $\log(x+4)$), so there is

$\boxed{\text{no solution}}$

49) $18^x = 4^{2x+1}$

a) Take ln of both sides: $\ln(18^x) = \ln(4^{2x+1})$

Then apply power rule of logs:

$x \ln 18 = (2x+1) \ln 4$

49) b) $x \ln 18 = 2x \ln 4 + \ln 4$

$-2x \ln 4 \quad -2x \ln 4$

$\rightarrow x \ln 18 - 2x \ln 4 = \ln 4$

$\rightarrow x(\ln 18 - 2\ln 4) = \ln 4$

$\rightarrow \boxed{x = \frac{\ln 4}{\ln 18 - 2\ln 4}}$

c) Plug into calculator: $\boxed{x \approx 0.5091}$

$\rightarrow$ 50) $3^{x+2} = 10^{x-1} \rightarrow (x+2)\log 3 = (x-1)\cancel{\log 10}^{1}$

$x \log 3 + 2\log 3 = x - 1$

$-x\log 3 \quad +1 \qquad -x\log 3 + 1$

$\rightarrow 2\log 3 + 1 = x - x\log 3 = x(1 - \log 3)$

$\rightarrow \boxed{x = \frac{2\log 3 + 1}{1 - \log 3}}$

$\rightarrow$ 54) $y = 5^{x+3} \longrightarrow x = 5^{y+3} \rightarrow \log_5 x = y + 3$

$\rightarrow \boxed{y = f^{-1}(x) = \log_5 x - 3 = \frac{\log x}{\log 5} - 3}$

55) $y = 4e^{x-2} - 11 \longrightarrow x = 4e^{y-2} - 11 \rightarrow x + 11 = 4e^{y-2}$

$\rightarrow \frac{x+11}{4} = e^{y-2} \longrightarrow y - 2 = \ln\left(\frac{x+11}{4}\right)$

$\rightarrow \boxed{y = f^{-1}(x) = \ln\left(\frac{x+11}{4}\right) + 2}$

57) $\frac{30}{12.32} = \frac{12.32\, e^{0.0247t}}{12.32}$

$\frac{30}{12.32} = e^{0.0247t} \to 0.0247t = \ln\left(\frac{30}{12.32}\right)$

$\to t = \frac{1}{0.0247} \ln\left(\frac{30}{12.32}\right) \approx 36.$

$t = 0 \to 1930$, so $\boxed{t = 36 \to 1966}$

58) Doubling an investment: $2P = P\left(1 + \frac{.045}{12}\right)^{12 \cdot t}$

$\to 2 = (1.00375)^{12t} \to \log 2 = 12t \log 1.00375$

$\to \boxed{t = \frac{1}{12} \frac{\log 2}{\log 1.00375} \approx 15.43 \text{ years}}$

60) To find the half-life:

$Pe^{-0.0289t} = \frac{1}{2}P \to e^{-0.0289t} = \frac{1}{2}$

$\to -0.0289t = \ln\left(\frac{1}{2}\right) \to \boxed{t = \frac{-\ln\left(\frac{1}{2}\right)}{0.0289}}$

$\to \boxed{t \approx 24 \text{ thousand years}}$

63) $I = \frac{E}{R}\left(1 - e^{-Rt/L}\right) \to \frac{IR}{E} = 1 - e^{-\frac{Rt}{L}}$

$\to e^{-Rt/L} = 1 - \frac{IR}{E} \to -\frac{Rt}{L} = \ln\left(1 - \frac{IR}{E}\right)$

$\to \boxed{t = -\frac{L}{R} \ln\left(1 - \frac{IR}{E}\right)}$

Section 4.2, 39a,c, 49

39a) $A(t) = 22000\, e^{0.015t}$

c) $\frac{30\,000}{22000} = \frac{22000\, e^{0.015t}}{22000}$

$\rightarrow \frac{15}{11} = e^{0.015t} \rightarrow \frac{\ln\left(\frac{15}{11}\right)}{0.015} = \frac{0.015t}{0.015}$

$\rightarrow t = \frac{\ln\left(\frac{15}{11}\right)}{0.015} \approx 20.68 \approx 21$

$\rightarrow 1994 + 21 = \boxed{2015}$

$\rightarrow$ 49) $f(t) = \frac{2000}{1 + 199e^{-0.12t}}$

a) $f(7) = \frac{2000}{1 + 199\, e^{-0.12 \cdot 7}} = 23$ people.

b) $\frac{1000}{1000} = \frac{\frac{2000}{1 + 199\, e^{-0.12t}}}{1000}$

$\rightarrow 1 = \frac{2}{1 + 199e^{-0.12t}} \quad (\cdot (1 + 199e^{-0.12t}))$

$\rightarrow 1 + 199\, e^{-0.12t} = 2$

$\rightarrow 199e^{-0.12t} = 1$

$\rightarrow e^{-0.12t} = \frac{1}{199} \rightarrow -0.12t = \ln\left(\frac{1}{199}\right)$

$\rightarrow t = \frac{\ln\left(\frac{1}{199}\right)}{-0.12} \approx$ <u>44 days</u>