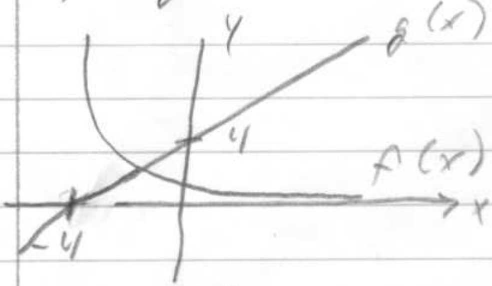
Section 4.3 4, 8, 9, 13, 14, 15, 18, 21, 24, 26, 29, 31, 34, 38, 45, 50, 51

4) $f(g(x)) = \frac{1}{(\frac{1}{x}+2)-2} = \frac{1}{\frac{1}{x}} = \underline{x}$

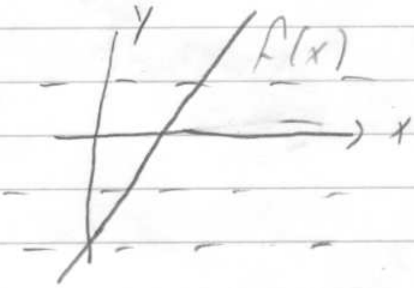
$g(f(x)) = \frac{1}{\frac{1}{x-2}} + 2 = x-2+2 = \underline{x}$

8) $f(g(x)) = \frac{1}{x+4+4} = \frac{1}{x+8} \neq x.$



Clearly are not reflections over $y=x$.

9)

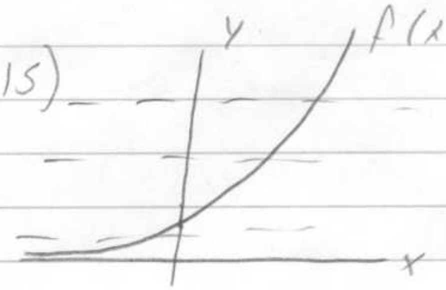


$f(x) = 2x - 7$ is one-to-one, passes horizontal line test.

13) $f(x) = x^3 + 3x - 2$ $f(x)$ is one-to-one.

14) $f(x) = \frac{2}{3}x^3 - 2x + 8$

$f(x)$ is not 1-to-1. y-values between 7 + 9 have more than one x-value

15) 

$f(x)=2^x$ is 1-to-1. It has an asymptote, but it passes the vertical line test.

18) $f(x)=12+\frac{3}{5}x \rightarrow x = 12 + \frac{3}{5}f^{-1}(x)$

$-12 \quad -12$

$\rightarrow x - 12 = \frac{3}{3} f^{-1}(x)$

$\cdot \frac{5}{3} \quad \cdot \frac{5}{3}$

$\rightarrow \boxed{f^{-1}(x) = \frac{5}{3}(x-12)}$

21) $f(x) = \frac{1}{x-5} \rightarrow x = \frac{1}{f^{-1}(x)-5}$

$\rightarrow \frac{x(f^{-1}(x)-5)}{x} = \frac{1}{x}$

$\rightarrow f^{-1}(x) - 5 = \frac{1}{x}$

$+5 \quad +5$

$\boxed{f^{-1}(x) = \frac{1}{x} + 5}$

24) $T(x) = \sqrt[3]{x} - 2 \rightarrow x = \sqrt[3]{T^{-1}(x)} - 2$

$+2 \quad +2$

$(x+2)^3 = \left(\sqrt[3]{T^{-1}(x)}\right)^3$

$\boxed{(x+2)^3 = T^{-1}(x)}$

26) $f(x) = x^2 - 3$

$f(x)$ is not 1-to-1.
To verify:

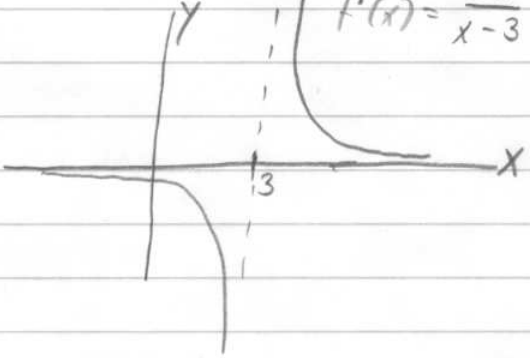
$f(c_1) = f(c_2) \rightarrow c_1^2 - 3 = c_2^2 - 3$
$\quad\quad\quad\quad\quad\quad\quad\quad +3 \quad\quad +3$

$\rightarrow c_1^2 = c_2^2$

$\rightarrow c_1 = \pm c_2$

$\rightarrow$ There are two solutions, so the function is not 1-to-1.

29) $f(x) = \frac{2}{x-3}$



$f(c_1) = f(c_2) \rightarrow \frac{2}{c_1 - 3} = \frac{2}{c_2 - 3}$ $\left(\cdot \frac{1}{2}\right)$

$\rightarrow \frac{1}{c_1 - 3} = \frac{1}{c_2 - 3}$ (take $\frac{1}{x}$ of both sides).

$\rightarrow c_1 - 3 = c_2 - 3$ $(+3)$

$\Rightarrow \underline{c_1 = c_2}$

$\Rightarrow$ Function is 1-to-1.

31) $f(x) = \frac{2x-1}{x+4} \rightarrow x = \frac{2y-1}{y+4}$ $(\cdot(y+4))$

$\rightarrow xy + 4x = 2y - 1$ $(-2y)$

$xy - 2y + 4x = -1$ $(-4x)$

$y(x-2) = -1 - 4x$ $(\div (x-2))$

$y = -\frac{1+4x}{x-2}$

$\rightarrow \boxed{f^{-1}(x) = \frac{4x+1}{2-x}}$

34) $f(x) = \frac{1}{\sqrt{x-3}} \rightarrow x = \frac{1}{\sqrt{y-3}}$ $(\cdot \sqrt{y-3})$

$\rightarrow x\sqrt{y-3} = 1$ $(\div x)$

$\sqrt{y-3} = \frac{1}{x}$ (square)

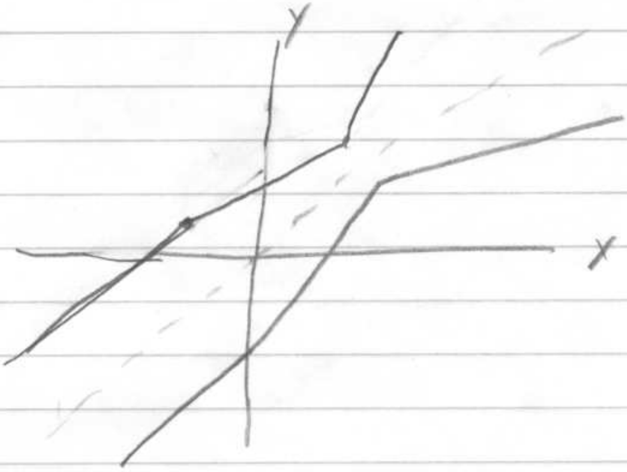
$y - 3 = \frac{1}{x^2}$ $(+3)$

$y = \frac{1}{x^2} + 3$.

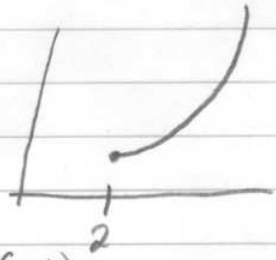
$\rightarrow \boxed{f^{-1}(x) = \frac{1}{x^2} + 3}$

Note: $f(x)$ has domain $x \geq 3$. and range $0 < y < \infty$.
Then $f^{-1}(x)$ has domain $0 < x < \infty$.

38)



45) $f(x) = x^2 - 4x + 5$
$= x^2 - 4x + (2)^2 - 4 + 5$
$= (x-2)^2 + 1$



for $x \geq h$, $x = (g(x)-2)^2 + 1$ (-1)
$x - 1 = (g(x)-2)^2$ $(\sqrt{\quad})$
* $\sqrt{x-1} = g(x) - 2$ $(+2)$
$\boxed{\sqrt{x-2} + 2 = g(x)}$

* I took only the positive square root here because we need the domain of f. $(x \geq 2)$ to match the range of g.

50) $C(n) = 28n + 5700 \rightarrow n = 28C^{-1}(n) + 5700$
$\rightarrow n - 5700 = 28C^{-1}(n)$

$\rightarrow \boxed{C^{-1}(n) = \frac{n-5700}{28}}$

$C(n)$ is the cost to manufacture n helmets.
$C^{-1}(n)$ gives the number of helmets that can be manufactured for a cost of n dollars.

51) $F = \frac{9}{5}c + 32 \rightarrow F - 32 = \frac{9}{5}c$

$\rightarrow \boxed{C = F^{-1}(f) = \frac{5}{9}(f-32)}$

F converts celsius to farenheit.
F^{-1} converts farenheit to celsius.