Problems 1-4 are meant to illustrate some common algebra mistakes that students might make. Imagine that a student attempted to simplify the expression in part a) and came up with the expression in part b). However, the student made a mistake. I will first show the correct steps that should have been taken, and describe the mistake that was made. The steps of algebra taken in each case are described to the right.

1. Recall that negative exponents are the same as taking the reciprocal, for example, $x^{-1} = \frac{1}{x}$, $x^{-2} = \frac{1}{x^2}$ or $\frac{1}{x^{-2}} = x^2$. To get from a) to b), you should do

$$\begin{aligned}
\text{Part a)} &= \frac{1}{3x^2} && \text{Split } 3x^2 \text{ into } 3 \cdot x^2 \\
&= \frac{1}{3 \cdot x^2} && \text{Then split into two fractions} \\
&= \frac{1}{3} \cdot \frac{1}{x^2} && \text{Rewrite } 1/x^2 \text{ using a negative exponent} \\
&= \frac{1}{3} x^{-2} && \text{Rewrite } 1/3 \text{ using a negative exponent} \\
&= 3^{-1} x^{-2}.
\end{aligned}$$

The mistake made in the workbook was not including the -1 exponent above the 3.

Part a) for $x = 4$ is given by

$$\begin{aligned}
\frac{1}{3(4)^2} &= \frac{1}{3 \cdot 16} \\
&= \frac{1}{48}
\end{aligned}$$

and part b) is given by

$$\begin{aligned}
3(4)^{-2} &= \frac{3}{4^2} \\
&= \frac{3}{16}.
\end{aligned}$$

Clearly these values are not the same.

2. The mistake was made in not dividing **both** terms in the numerator by 3. When we write an expression like $\frac{6a+1}{3}$ we put all of $6a + 1$ in the numerator because we want it to be treated like a single number. You need to divide both terms by 3 because you need to remove the denominator from the “whole” number in the numerator, not just a part. The correct steps would be

$$\begin{aligned}
\text{Part a)} &= \frac{6a+1}{3} && \text{Split the fraction at the addition symbol} \\
&= \frac{6a}{3} + \frac{1}{3} && \text{Pull the } a \text{ out of the first fraction} \\
&= \frac{6}{3} a + \frac{1}{3} && \text{Simply } 6/3 \\
&= 2a + \frac{1}{3}.
\end{aligned}$$

Getting $2a + 1$ means that you did not divide both terms on top by 3.

Evaluating for $x = -1$ in part a) gives

$$\begin{aligned}
\frac{6(-1)+1}{3} &= \frac{-6+1}{3} \\
&= \frac{-5}{3}
\end{aligned}$$

and evaluating in part b) gives

$$\begin{aligned} 2(-1)+1 &= -2+1 \\ &= -1 \end{aligned}$$

3. The mistake made in this problem is not carrying the exponent through to all terms. There are multiple ways to approach this problem, so I will present two:

$$\begin{aligned} \text{Part a)} &= (-3x)^3 && \text{Distribute the exponent} \\ &= (-3)^3 \cdot x^3 && \text{Expand } (-3)^3 = (-3)\cdot(-3)\cdot(-3) \\ &= (-3)\cdot(-3)\cdot(-3)\cdot x^3 && \text{Evaluate the multiplication} \\ &= -27\cdot x^3 && \text{Simplify just a little} \\ &= -27x^3 \end{aligned}$$

and the second method:

$$\begin{aligned} \text{Part a)} &= (-3x)^3 && \text{Distribute the exponent} \\ &= (-3)^3 \cdot x^3 && \text{Write } -3 = -1\cdot 3 \text{ and distribute the exponent again} \\ &= (-1)^3 \cdot 3^3 \cdot x^3 && \text{Evaluate } (-1)^3 \text{ and } 3^3 \\ &= -1\cdot 27\cdot x^3 && \text{Carry out the multiplications} \\ &= -27x^3. \end{aligned}$$

Evaluating for $x = 2$ in part a) gives

$$\begin{aligned} (-3\cdot 2)^3 &= (-6)^3 \\ &= -216 \end{aligned}$$

and evaluating part b) gives

$$\begin{aligned} -3(2)^3 &= -3\cdot 8 \\ &= -24 \end{aligned}$$

4. The mistake in this problem is not keeping track of the minus signs. The correct steps are

$$\begin{aligned} \text{Part a)} &= \frac{z-1}{z-2} && \text{Factor a minus sign out of the numerator} \\ &= \frac{-(-z+1)}{z-2} && \text{Re-arrange the part in parentheses} \\ &= \frac{-(1-z)}{z-2} && \text{Factor a minus sign out of the denominator} \\ &= \frac{-(1-z)}{-(-z+2)} && \text{Re-arrange the parentheses in the denominator} \\ &= \frac{-(1-z)}{-(2-z)} && \text{Re-write } -(1-z) = -1\cdot(1-z)\text{, and the same for the denominator} \\ &= \frac{-1\cdot(1-z)}{-1\cdot(2-z)} && \text{Split into two fractions at the multiplication sign} \\ &= \frac{-1}{-1}\cdot\frac{1-z}{2-z} && \text{Notice that } \tfrac{-1}{-1} = 1 \\ &= \frac{1-z}{2-z} \end{aligned}$$

Evaluating part a) for $z = 3.5$ gives

$$\begin{aligned}\frac{3.5-1}{3.5-2} &= \frac{2.5}{1.5}\\ &= \frac{\frac{5}{2}}{\frac{3}{2}}\\ &= \frac{\frac{5}{2}}{\frac{3}{2}} \cdot \frac{2}{2}\\ &= \frac{5}{3}\end{aligned}$$

and evaluating part b) gives

$$\begin{aligned}-\frac{1-3.5}{2-3.5} &= -\frac{2.5}{1.5}\\ &= -\frac{\frac{5}{2}}{\frac{3}{2}}\\ &= -\frac{\frac{5}{2}}{\frac{3}{2}} \cdot \frac{2}{2}\\ &= -\frac{5}{3}\end{aligned}$$

5. $f(x) = -x^2 + 5x - 4$.

$$\begin{aligned}f(1) &= -(1)^2 + 5(1) - 4\\ &= -1 + 5 - 4\\ &= 0\end{aligned}$$

A very common mistake was made in evaluating the first term of f, $-(1)^2$. Some students wrote $-(1)^2 = 1$, some dropped the parentheses and wrote $-1^2 = 1$, and some brought the minus sign inside the parentheses and wrote $(-1)^2 = 1$. Each of these shows that the equation is being misinterpreted. We can avoid these mistakes by realizing that $x^2 = x \cdot x$, which gives $f(x) = -x \cdot x + 5x - 4$. Then remember that $-x = -1 \cdot x$, which gives $f(x) = -1 \cdot x \cdot x + 5x - 4$. Evaluating this expression gives

$$\begin{aligned}f(1) &= -1 \cdot (1) \cdot (1) + 5(1) - 4\\ &= -1 \cdot 1 + 5 - 4\\ &= -1 + 5 - 4\\ &= 0.\end{aligned}$$

6. $g(a) = \frac{a+1}{a-1}$.

$$\begin{aligned}g(1) &= \frac{1+1}{1-1}\\ &= \frac{2}{0}\end{aligned}$$

The solution is not defined, therefore we know that $a = 1$ is not in the domain of g.

7. $h(y) = 6y^0 - (6y)^0$

$$\begin{aligned}h(10) &= 6(10)^0 - (6 \cdot 10)^0\\ &= 6 \cdot 1 - (60)^0\\ &= 6 - 1\\ &= 5\end{aligned}$$

8. $k(x) = \frac{x^2+62.5}{0.1x+3}$.

$$\begin{aligned} k(10) &= \frac{10^2 + 62.5}{0.1 \cdot 10 + 3} \\ &= \frac{100 + 62.5}{1 + 3} \\ &= \frac{162.5}{4} \end{aligned}$$

9. **a)** I will give two methods of solving this. First

$$\begin{aligned} 2 - 3x &\geq 0 && \text{Subtract 2 from both sides} \\ -3x &\geq -2 && \text{Divide both sides by } -3\text{. Make sure to flip the inequality sign} \\ x &\leq \frac{2}{3} \end{aligned}$$

and second

$$\begin{aligned} 2 - 3x &\geq 0 && \text{Add } 3x \text{ to both sides} \\ 2 &\geq 3x && \text{Divide both sides by 3} \\ \frac{2}{3} &\geq x \end{aligned}$$

b)

$$\begin{aligned} 3x + 4 &\leq 0 && \text{Subtract 4 from both sides} \\ 3x &\leq -4 && \text{Divide both sides by 3} \\ x &\leq -\frac{4}{3} \end{aligned}$$

c) This problem is quite tricky to do algebraically, but much easer to do graphically. First, I will find the areas that are satisfied by an equality

$$\begin{aligned} 4 - x^2 &= 0 && \text{Add } x^2 \text{ to both sides} \\ 4 &= x^2 && \text{Take the square root. Remember the } \pm \\ \pm 2 &= x \end{aligned}$$

so the points $x = 2$ and $x = -2$ both satisfy the inequality. These points split the real line into three parts: $(-\infty, -2)$, $(-2, 2)$ and $(2, +\infty)$. We will take points from each of these three parts and test them against the inequality.

$(-\infty, -2)$**:** Let's check $x = -3$

$$\begin{aligned} 4 - (-3)^2 &= 4 - 3^2 \\ &= 4 - 9 \\ &= -5 < 0 \end{aligned}$$

so this interval does not satisfy the inequality.

$(-2, 2)$**:** Let's check $x = 0$

$$4 - (0)^2 = 4 - 0 = 4 > 0$$

so this interval satisfies the inequality.

$(2, +\infty)$**:** Let's check $x = 5$

$$4 - (5)^2 = 4 - 25 = -21 < 0$$

so this interval does not satisfy the inequality.

Combining all this, we know that the inequality is satisfied by $x = -2$, $x = 2$ and the interval $(-2, 2)$. Thus the solution is $-2 \leq x \leq 2$.

10. Recall that the domain of a function is the set of inputs for which the function has an output. Here we have two problems involving square roots. We know that you cannot take the square root of a negative number (e.g., $\sqrt{-9}$ doesn't exist), so we must find the areas where the expression under the square root is positive or zero.

a) To find the domain we must find where the part under the radical is positive or zero. Thus

$$\begin{aligned} 5 - 2x &\geq 0 && \text{Add } 2x \text{ to both sides} \\ 5 &\geq 2x && \text{Divide both sides by 2} \\ \frac{5}{2} &\geq x \end{aligned}$$

so the domain is $x \leq \frac{5}{2}$, or, in interval notation, $(-\infty, \frac{5}{2}]$.

b) Again, we must find where the part under the radical is positive or zero.

$$\begin{aligned} 2x - 9 &\geq 0 && \text{Add 9 to both sides} \\ 2x &\geq 9 && \text{Divide both sides by 2} \\ x &\geq \frac{9}{2} \end{aligned}$$

So the domain is $\frac{9}{2} \leq x$, or, in interval notation, $[\frac{9}{2}, +\infty)$.