3/9/2011
Section 10

<u>Vertex formula</u>: Given $y = ax^2 + bx + c$,
the vertex (h, k) can be found by

$$h = \frac{-b}{2a}$$

$$k = \text{plug in } \underset{\text{"}x\text{"}}{\frac{-b}{2a}} \text{ to formula}$$

<u>Example</u>: Find the vertex:

ⓐ $f(x) = 2x^2 + 4x + 9$

$$h = -\frac{b}{2a} = -\frac{4}{2(2)} = -1$$

$$k = 2(-1)^2 + 4(-1) + 9 = 2 - 4 + 9 = 7$$

$$\Rightarrow (-1, 7) \leftarrow \text{min}$$

ⓑ $g(m) = -\frac{1}{2}m^2 - 3m + 1$

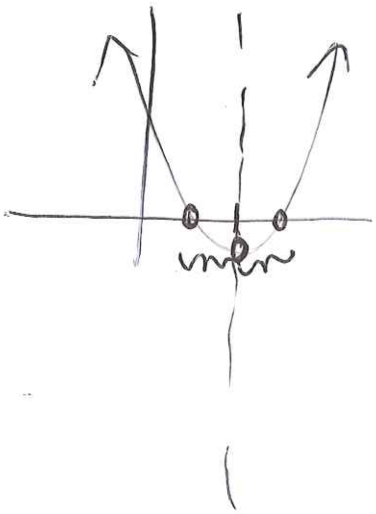
$$h = \frac{-(-3)}{2(-\frac{1}{2})} = -3$$

$$k = -\frac{1}{2}(-3)^2 - 3(-3) + 1 = -\frac{1}{2}9 + 9 + 1 = 5.5 = \frac{11}{2}$$

$$\Rightarrow (-3, \tfrac{11}{2})$$
max

Fact / observation:

parabolas are symmetric about the vertical line thru the vertex

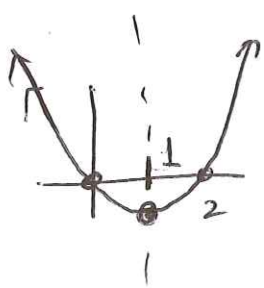


X-

if it's easy to find the intercepts, then we can use symmetry to find the vertex.

Example 2:

ⓐ $y = x^2 - 2x = x(x-2)$



x int: 0, 2

↑

horiz value of vertex is just the avg of x-int: $1 = \frac{0+2}{2}$

vertical value: plug in 1 for x: $1^2 - 2(1) = -1$

$\Rightarrow (+1, -1)$

(b) $y = 2x^2 - x - 1$

$= (2x + 1)(x - 1)$

$x = -\frac{1}{2}, 1$

$h = \frac{-\frac{1}{2} + 1}{2} = \frac{1}{4}$

$k = \underbrace{(2(1/4) + 1)}_{\substack{1.5 \\ \frac{3}{2}}} \underbrace{(\tfrac{1}{4} - 1)}_{\substack{0.75 \\ \frac{3}{4}}} = \frac{9}{8}$

$(\frac{1}{4}, 9/8)$

What of 2.5 is expected for test?

- intercepts! (old news)
- Finding vertex
- sketch graph / finding max, min.

Example 3

- Draw an accurate sketch of $y = 2x^2 - x - 1$

xint, yint, vertex, label if max/min.

- yint: $(0, 2(0)^2 - 0 - 1 = -1)$ $(0, -1)$
- xint: $2x^2 - x - 1 = 0$ $(-1/2, 0)$

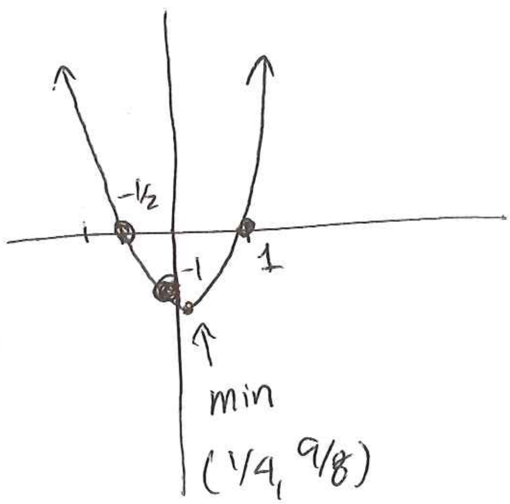
$(2x + 1)(x - 1) = 0$ $(1, 0)$

$x = -1/2, 1.$

∘ vertex: $h = -b/2a = -(-1)/2(2) = 1/4$

$k = (\text{plug in } x = 1/4) = 9/8$

$(1/4, 9/8)$



2.) ⓑ $(x-1)(x+1) - 20x$

$= x^2 - 1 - 20x = x^2 - 20x - 1$

Teacher is crazy
Doesn't factor
nicely!

Headache way to factor
Q.F.

$$x = \frac{20 \pm \sqrt{20^2 + 4(1)(+1)}}{2}$$

x int.

$$= \frac{20 \pm \sqrt{404}}{2} = \frac{20 \pm 2\sqrt{101}}{2} = 10 \pm \sqrt{101}$$

$\rightarrow (x - (10 + \sqrt{101}))(x + (10 - \sqrt{101}))$

$(x+a)^2 = x^2+2ax+a^2$ 2/9 Section 12

Warm up: Complete the square & find the vertex:

ⓐ $f(x) = (x^2+4x)+2$

$= \left(\underbrace{x^2+4x+\left(\tfrac{4}{2}\right)^2}-\left(\tfrac{4}{2}\right)^2\right)+2$

$= (x+2)^2 - 4 + 2$

$= (x+2)^2 - 2$ vertex: $(-2,-2)$

ⓑ $g(t) = (-t^2+8t)+1$

$= -\left(\underbrace{t^2-8t+\left(\tfrac{-8}{2}\right)^2}-\left(\tfrac{-8}{2}\right)^2\right)+1$ $(-4)^2$

$= -\left((t-4)^2-16\right)+1$

$= -(t-4)^2+16+1$ vertex: $(4,17)$

$= -(t-4)^2+17$

ⓒ $f(p) = (4p^2+16p)+1$

$= 4(\underbrace{p^2+4p+4}-4)+1$

$= 4((p+2)^2-4)+1$

$= 4(p+2)^2-16+1$

$= 4(p+2)^2-15$

vertex: $(-2,-15)$

ⓓ $y = ax^2+bx+c$

$= a\left(\underbrace{x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2}-\left(\frac{b}{2a}\right)^2\right)+c$

$= a\left(\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}\right)+c$

$= a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c$

vertex: $\left(\frac{-b}{2a},\ \text{gross}\right)$

Part d gives us an alternate way to find the vertex!

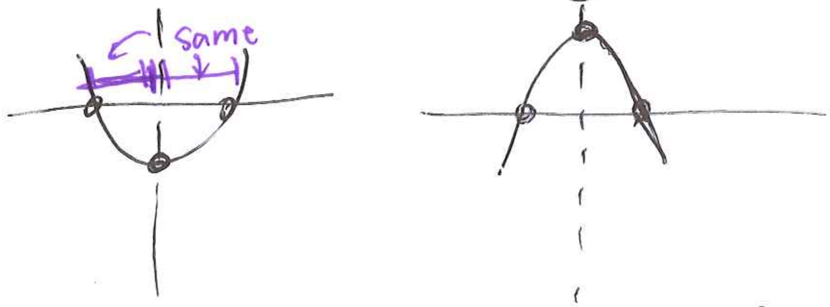
In general, given parabola $y = ax^2+bx+c$ the vertex (h,k) can be found by

$h = \frac{-b}{2a}$, $k =$ plug $x = \frac{-b}{2a}$ into my formula for y.

vertex formula

Another way to find the vertex:

Observe a parabola is symm about the vertical line passing thru its vertex



Given the x intercepts (if any) the horiz coord of vertex h is simply the average of these.

Example: Find the vertex exactly

ⓐ $y = 2x^2 - x - 1$

$= (2x+1)(x-1)$

x int : $-1/2, 1$

vertex: $h = \frac{-1/2 + 1}{2} = \frac{1}{4}$

$k = \underbrace{(2(\tfrac{1}{4})+1)}_{3/2}\underbrace{(\tfrac{1}{4}-1)}_{-3/4} = -\frac{9}{8}$

$(\frac{1}{4}, -\frac{9}{8})$

ⓑ $y = x^2 + x + 1$

Doesn't factor!

Easier to use comp square or formula.

$h = -\frac{1}{2}$

$k = (-\frac{1}{2})^2 + (-\frac{1}{2}) + 1$

$\frac{1}{4} - \frac{1}{2} + 1$

$= 3/4$

$(-\frac{1}{2}, 3/4)$

For test: | §2.5 :

- intercepts (old news)
- Finding vertex (use one of 3 techniques)
- Graphs.

Example 2: Graph $y = 2x^2 - x - 1$.

intercepts, vertex, label if max/min.

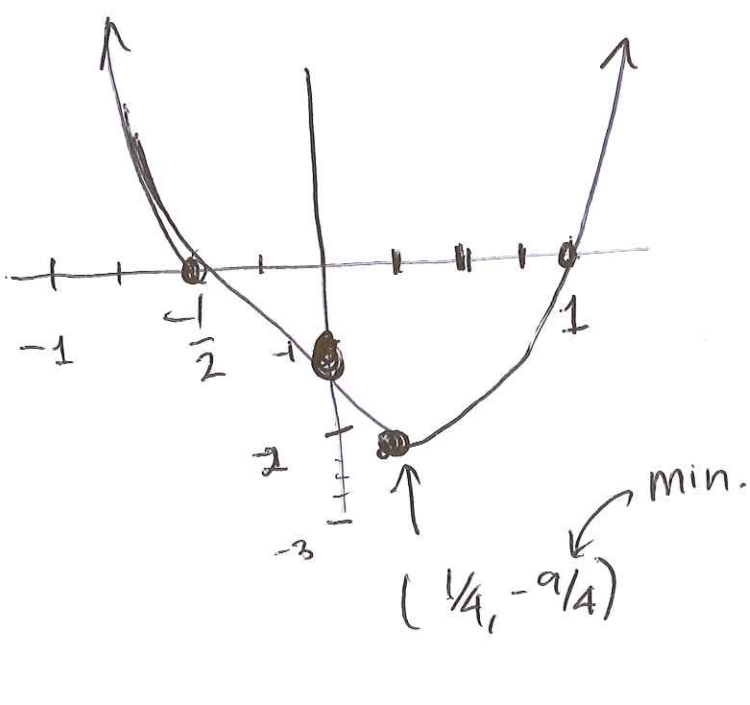
Intercepts: y int: $(0, -1)$

x int: $0 = 2x^2 - x - 1$

$0 = (2x+1)(x-1) \rightarrow (-\tfrac{1}{2}, 0) \quad (1, 0)$

vertex: $h = -b/2a = \dfrac{-(-1)}{2(2)} = \underline{1/4}$

$k = 2(1/4)^2 - 1/4 - 1 = -9/4$



2 (b) on Review is incorrect!

$(x-1)(x+1)-20x = x^2-1-20x = x^2-20x-1$

ugly factorization:

$(x-a)$ is a factor $\iff$ $x=a$ is a zero

e.g. $x-2$ " $\iff$ $x=2$ is a zero.

Watch signs

Q.F. $x = \dfrac{20 \pm \sqrt{400+4}}{2} = \dfrac{20 \pm \sqrt{404}}{2} = 10 \pm \sqrt{101}$

$\Rightarrow x^2-20x-1 = (x-(10+\sqrt{101}))(x-(10-\sqrt{101}))$

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§1.7

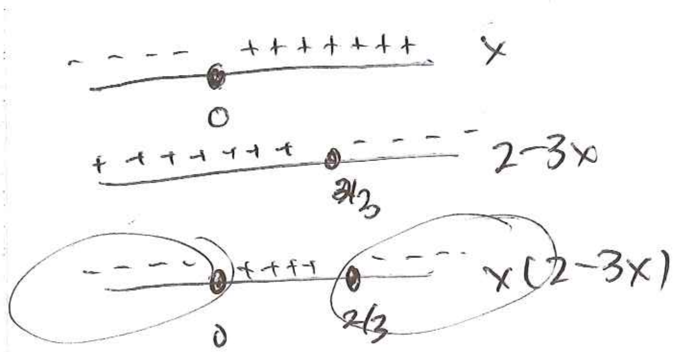
negative or zero.

#32.) $x(2-3x) \le 0$

$+ \quad -$

$- \quad +$

$(-\infty, 0] \cup [2/3, \infty)$



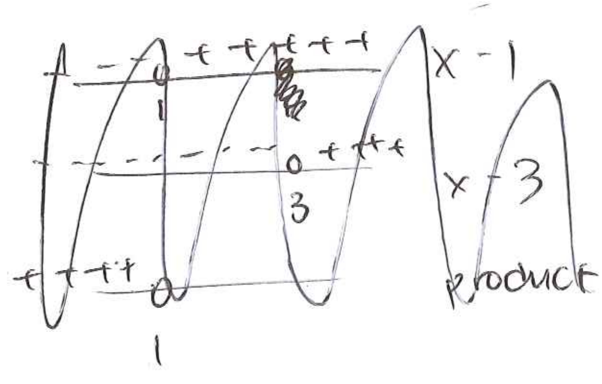
#72.)

#40.)

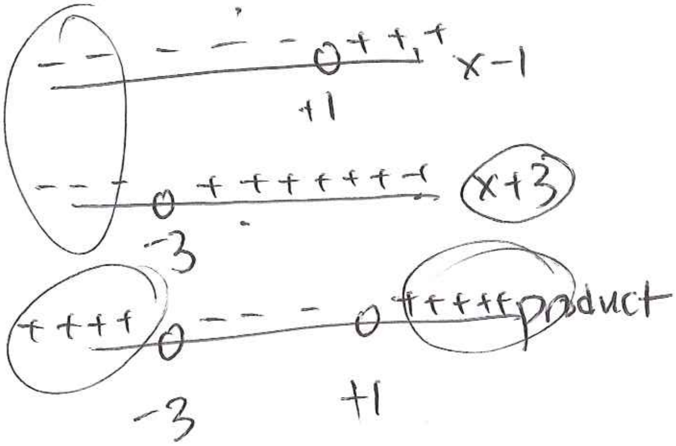
$x^2+2x > 3$

$x^2+2x-3 > 0$

$(x-1)(x+3) > 0$



$(x-1)(x+3) > 0$ pos.



$(-\infty, -3) \cup (+1, \infty)$

#72.) $\left|\frac{x+1}{2}\right| \geq 4$

$|x+1| \geq 8$

$x+1 \geq 8$ or $x+1 \leq -8$

$-1 \quad -1 \qquad\qquad -1 \quad -1$

$x \geq 7$ or $x \leq -9$

$(-\infty, -9] \cup [7, \infty)$