4/20/2011 W
Section 12

§7.3 Some formulas are
Not covered on Exam 4, Final Exam

DO NOT need to know : Half-Angle, Sum → Product, or Product → Sum.

DO NEED TO KNOW : Double Angle ← will be provided
Power Reduction ← will not be provided.

Double Angle:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

Example: Suppose $\cos x = -2/3$, x in QII.

Find $\cos(2x)$, $\sin(2x)$, $\tan(2x)$.

$$\cos(2x) = 2\cos^2\theta - 1 = 2(-2/3)^2 - 1 = 8/9 - 1 = \boxed{-1/9}$$

$$\sin(2x) = 2\sin x\cos x = 2\left(\sqrt{5}/3\right)\left(-2/3\right) = \boxed{\frac{-4\sqrt{5}}{9}}$$

need to find this to calculate $\sin(2x)$

$$\sin^2 x + \overset{(-2/3)^2}{\cos^2 x} = 1$$

$$\sin^2 x = 1 - 4/9$$

$$\sin^2 x = 5/9$$

$$\sin x = \sqrt{5}/3$$

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{-4\sqrt{5}/9}{-1/9} = \boxed{4\sqrt{5}}$$

Q: Is there a similar formula for $\sin(3\theta)$, $\sin(4\theta)$, ... etc.

A: NOT NEARLY AS NICE.

↑ NOT on Final.

Power Reduction formulas: (not provided on final)

└ Reduce power.

Deriving these:

$$\cos(2\theta) = 2\cos^2\theta - 1$$
$$\frac{\cos(2\theta)+1}{2} = \frac{2\cos^2\theta}{2}$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$
$$\frac{\cos(2\theta)-1}{-2} = \frac{-2\sin^2\theta}{-2}$$

FORMULAS:

$$\cos^2\theta = \frac{\cos(2\theta)+1}{2} = \frac{1+\cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

Example

Reduce / Simplify using Double angle identities. (So no exponents $\neq 1$)

ⓐ $\sin^2 x\cos^2 x = \left(\frac{1-\cos 2\theta}{2}\right)\left(\frac{1+\cos 2\theta}{2}\right)$

$$= \frac{1}{4}(1-\cos^2 2\theta)$$

↓ pyth

$$= \frac{1}{4}(\sin^2 2\theta)$$

↓ Power Reduction

$$= \frac{1}{4}\left(\frac{1-\cos(2(2\theta))}{2}\right)$$

$$= \frac{1}{8}(1-\cos 4\theta)$$

ⓑ $\cos^4 x = (\cos^2 x)^2 = \left(\frac{\cos(2\theta)+1}{2}\right)^2$

further reduce

$$= \frac{1}{4}\left(\cos^2\boxed{2\theta} + 2\cos(2\theta) + 1\right)$$

$$= \frac{1}{4}\left(\frac{\cos(4\theta)+1}{2} + 2\cos(2\theta) + 1\right)$$

Practice Solutions:

1.) (a) $$\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1} = \frac{\sec x + 1 + \sec x - 1}{\sec^2 x - 1}$$

$$\frac{\sin^2 x + \cos^2 x = 1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$= \frac{2\sec x}{\tan^2 x}$$

$$= \frac{2\frac{1}{\cos x}}{\frac{\sin^2 x}{\cos^2 x}}$$

$$= \boxed{\frac{2\cos x}{\sin^2 x}}$$

(b) $$\frac{\cos^3 x + \sin^2 x \cos x}{\tan x} = \frac{\cos x(\overbrace{\cancel{\cos^2 x + \sin^2 x}}^{1})}{\tan x}$$

$$= \frac{\cos x}{\frac{\sin x}{\cos x}} = \boxed{\frac{\cos^2 x}{\sin x} = \cot x \cdot \cos x}$$

(c) $$\frac{\sin 2\theta + \cos 2\theta - 1}{\sin\theta} = \frac{2\sin\theta\cos\theta + (\cancel{1} - 2\sin^2\theta) - \cancel{1}}{\sin\theta}$$

$$= \frac{2\cancel{\sin\theta}(\cos\theta - \sin\theta)}{\cancel{\sin\theta}}$$

$$= \boxed{2(\cos\theta - \sin\theta)}$$

$$= \frac{1}{8}\left(\cos(4\theta) + 1 + 4\cos 2\theta + 2\right)$$

$$= \frac{1}{8}\left(\cos(4\theta) + 4\cos(2\theta) + 3\right)$$

You try: (for notes)

Practice

1.) Simplify completely

(a) $\dfrac{1}{\sec x - 1} + \dfrac{1}{\sec x + 1}$

(b) $\dfrac{\cos^3 x + \sin^2 x \cos x}{\tan x}$ Hint: FACTOR top.

(c) $\dfrac{\sin 2\theta + \cos 2\theta - 1}{\sin\theta}$

2.) Evaluate $\tan 2x$ if $\sec x = 3$ and x in Q IV.

3.) Reduce the power:

$\sin^4 x$

2.) $\tan 2x$ if $\sec x = 3$ in QIV.

$$\tan 2x = \frac{\sin 2x}{\cos 2x}.$$

$\frac{1}{3} = \cos x$

$\Rightarrow \sin(x) = ?$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + (1/3)^2 = 1$

$\sqrt{\sin^2 x} = \sqrt{8/9}$

$\sin x = \pm \sqrt{8}/3$

$\Rightarrow$ QIV

$\sin x = -\sqrt{8}/3$

$$\sin 2x = 2 \sin x \cos x$$

$$= 2\left(\frac{-\sqrt{8}}{3}\right)\left(\frac{1}{3}\right)$$

$$= -\frac{2\sqrt{8}}{9} = \frac{-4\sqrt{2}}{9}$$

$$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{3}\right)^2 - 1 = \frac{2}{9} - 1 = 7/9$$

$$\Rightarrow \tan 2x = \frac{\sin(2x)}{\cos(2x)} = \frac{-\frac{4\sqrt{2}}{9}}{7/9} = \frac{-4\sqrt{2}}{7}$$

3.) Next time!

$\sin^4 x =$