7:45am ILC 120 Section 12

- catcard
- graphing calculator
  - ↳ eval
  - ↳ quad
- pencil.
- Can't leave until 9AM.
  Ends @ 10AM

I.) Basic Concepts

Defin of function:
Each input corresponds to at most one output.

Ex Determine if y is a function of x.

(a)

x	1	2	3	1
y	1	1	1	1

1 → 1
1 → 1
no problem

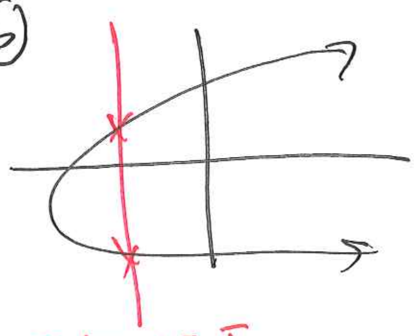
⇒ Function

x	1	2	3	1
y	1	2	3	4

1 → 1
1 → 4
2 diff outputs

⇒ not a function.

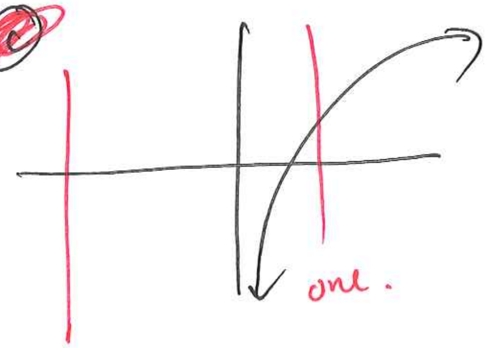
(b)



Fails VLT

⇒ NOT

(c)



one.

none

Passes VLT.

⇒ Function

(c) $x^2+y=5$

$y=5-x^2$

Yes, function.

$x+y^2=5$

$\sqrt{y^2}=\sqrt{5-x}$

$|y|=\sqrt{5-x}$

$y=\pm\sqrt{5-x}$

No, not function.

Domain: all inputs that can be eval in the function.

Range: all outputs that come out of the function.

Domains Restricted

- fractional expressions $\frac{n(x)}{d(x)}$

  $d(x) \neq 0$.

- square (even) roots $\sqrt{f(x)}$

  $f(x) \geq 0$

- log: $\log_b(f(x))$

  $f(x) > 0$

- SOME trig functions:

  $\tan\theta = \frac{\sin\theta}{\cos\theta}$ ← denom can't be zero

  $\sec\theta = \frac{1}{\cos\theta}$ ← denom can't be zero.

Some inv trig functions

$\sin^{-1}(y)$

↑ vert val on circle.

Linear functions   Arc (Avg Rate of Change)

$y = mx + b$ slope intercept
m $(0, b)$

$y = y_1 + m(x - x_1)$ point slope
(x_1, y_1) m

$m = \frac{\Delta y}{\Delta x} = \frac{\Delta \text{out}}{\Delta \text{in}} = \frac{\text{rise}}{\text{run}}$;   m units = $\frac{\text{units of } y}{\text{units of } x}$.

To find linear function:

1.) Find slope.
2.) use slope int   + a point on line
or point slope

$\Rightarrow$ get an equation.

A.R.C. $\rightsquigarrow$ linear:
linear is a function of constant slope / ARC.
ARC
Slope of $f(x)$ over $[a, b]$.

$= \frac{\Delta \text{out}}{\Delta \text{in}} = \frac{f(b) - f(a)}{b - a}$

Quads

$y = ax^2 + bx + c$ general

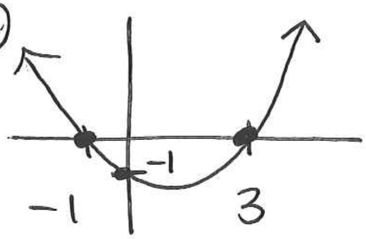
$y = a(x-h)^2 + k$ vertex (h, k)

$y = a(x - r_1)(x - r_2)$ factored. r_1, r_2 are zeros/x-int.

$\underline{a>0}$ $\underline{a<0}$

Example: Find the equation:

(a)



$\Rightarrow y = a(x-r_1)(x-r_2)$

$y = a(x+1)(x-3)$

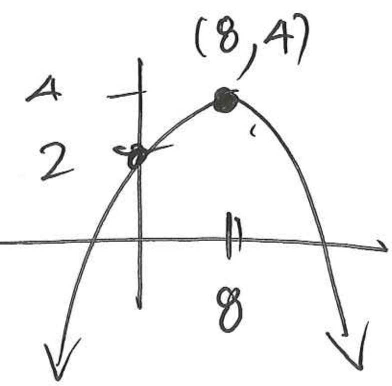
To find "a":

$-1 = a(0+1)(0-3)$

$\frac{1}{3} = a$

$\Rightarrow y = \frac{1}{3}(x+1)(x-3)$

(b)



$y = a(x-h)^2 + k$

$y = a(x-8)^2 + 4$

To find: "a":

$2 = a(0-8)^2 + 4$

$-2 = a(64)$

$\rightarrow a = \frac{-1}{32}$

$\boxed{y = \frac{-1}{32}(x-8)^2 + 4}$

Applications: Find max ← $a<0$

Find min ← $a>0$

occurs @ vertex (h, k)

minimizing / maximizing

minimum / maximum.

Polynomials:

$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \quad \text{general}$$

$$= a_n (x - r_1) \cdots (x - r_m) \quad \text{factored.}$$

degree: highest power of x that appears. (number)

lead coeff: the coeff in front of the highest power of x (number)

lead term: the term w/ largest power of x.

$$y = 2x^3 - 49x^{100} + 202x^5$$

deg 100

L.C. -49

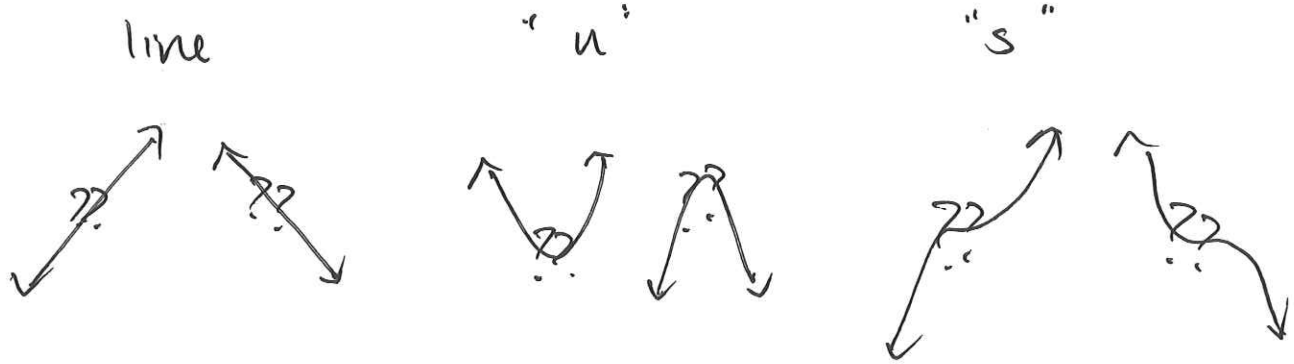
Lead term $-49x^{100}$.

Poly Facts: For poly of degree n

① # of zeros $\leq n$

② # of Turning points $\leq n-1$

③ lead term prop: $p(x)$ and its lead term have same ex beh:



④ Graph prop:

- continuous
- smooth
- domain all reals w/ ex beh as above.

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⑤ Factor Thm (For poly)

If $f(c)=0$, then $x-c$ is a factor of $f(x)$

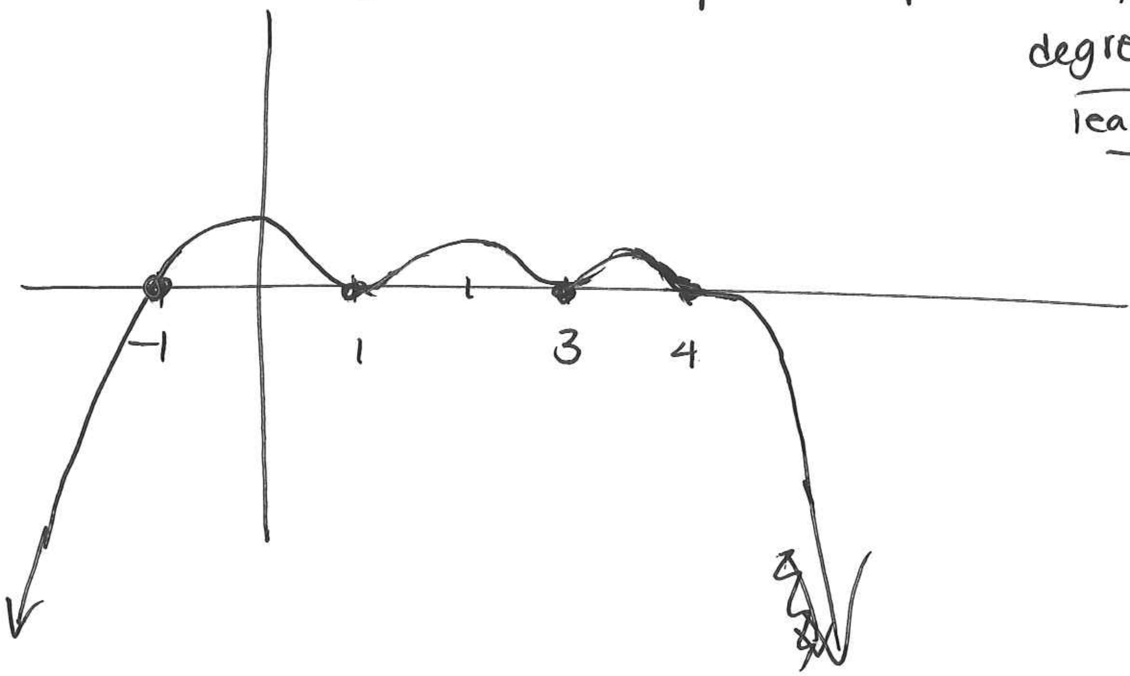
multiplicity: describes the type of intersection w/ x-axis.

Ex. Sketch $y = -2(x-1)^2(x+1)(x-3)^4(x-4)^5$

degree? 12

lead coef: -2

$-2x^{12}$



Polynomials:

Graph properties

- continuous – no breaks or jumps.
- smooth – no corners or cusps.
- must have extreme behavior of one of the following:

  line, "S", or "u"

  as $x \to \infty$, $y \to \pm\infty$

  as $x \to -\infty$, $y \to \pm\infty$

  $\boxed{\text{OR}}$ graph could be constant.

Rational function:

$$f(x) = \frac{n(x)}{d(x)};$$ $n(x), d(x)$ must be polynomial.

Domain: $d(x) \neq 0$
- VA: reduced denominator.
- holes: common factors/zeros.

Zeros: zeros of reduced numerator.

Ex beh: $\frac{n(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$ long division

$\frac{n(x)}{d(x)}$ has ex beh as $q(x)$.

Shortcut:

HA @ $y=0$: deg n < deg d

HA @ $y=$ ratio of lead coeff : deg n = deg d

SA @ $y=$ quotient: deg n = deg d + 1

Special cases! If deg n > deg d, no linear asymptote.

Exponentials: Compounding Interest

$y = P(1+\frac{r}{n})^{nt}$ given on final — compounded n-times.

$y = Pe^{rt}$ given on final — "compounded CONTINUOUSLY" "relative"

$y = Ce^{kt}$

continuous "relative" exponential.

$k=$ relative rate;

continuous rate.

$k>0$ growth

$k<0$ decay.

$y = Ca^{t}$

general exponential

$a=$ growth/decay factor.

rate $= |a-1|$

$a>1$ growth

$a<1$ decay.

Other formulas like logistic growth, log scales of pH/Richter, Newton's Law will be provided.

Inverses:

Original:	Inverse:
Input →	Output
Output →	Input.

" Inverse undoes the original"

Shortcuts to finding inverses:

Table: Switch inputs/outputs

Graph: Reflect over $y = x$

Formula: solve for the input; switch variables y $f^{-1}(x)$.

When is a function invertible?

Only when it has the prop ~~one~~ one-to-one: for each output there is at most one input.

HLT

Logarithms:

$$\log_b(AB) = \log_b A + \log_b(B)$$

$$\log_b(A/B) = \log_b A - \log_b B$$

$$\log_b(A^p) = p\log_b A.$$

$$\log_b(1) = 0$$

$$\log_b(b) = 1$$

$\log_b$ cancels $b^\wedge$

$$\log_b(b^x) = x$$

$$b^{\log_b x} = x$$

Transformations (Ch. 2?)

$$af(bx \pm c) + d$$

① outside output (same!) mult y-values by a

② inside input (opposite) divide x-val by b

① inside input (opposite) subtract c from x-vals

② outside output (same!) add to y-vals.

Trig — look at notes for Exam 4

$S = r\theta$ arclength, θ in rad

$A = \frac{1}{2} r^2 \theta$ area of sector; θ in rad

$\omega = \frac{\theta}{t}$ angular velocity θ any units, t = time

$v = \frac{s}{t}$ linear velocity s = distance, t = time

- Reduce $\cos^4 t$

$(\cos^2 t)^2$

$= \left(\frac{1+\cos 2t}{2}\right)^2$

$= \frac{1}{4}\left(1 + 2\cos 2t + \cos^2(2t)\right)$

$= \frac{1}{4}\left(1 + 2\cos 2t + \frac{1+\cos 4t}{2}\right)$

$= \frac{1}{8}\left(3 + 4\cos 2t + \cos 4t\right)$

- Reduce $\sin^4 t$

$(\sin^2 t)^2$

$= \left(\frac{1-\cos(2t)}{2}\right)^2$

$= \frac{1}{4}\left(1 - 2\cos(2t) + \cos^2 2t\right)$

$= \frac{1}{4}\left(\overset{2}{1} - 2\cos(2t) + \frac{1+\cos(4t)}{2}\right)$

$= \frac{1}{8}\left(3 - 4\cos 2t + \cos(4t)\right)$

$\cos(2t) = 2\cos^2 t - 1$

$\frac{1+\cos(2t)}{2} = \cos^2 t$

$\cos(2t) = 1 - 2\sin^2 t$

$\frac{1-\cos(2t)}{2} = \sin^2 t$

Solve for $\sin^2 t$ or $\cos^2 t$

- Find all exact solutions $0 \le \theta \le 2\pi$

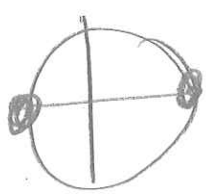
$\cos(2\theta) + \sin\theta = 1$

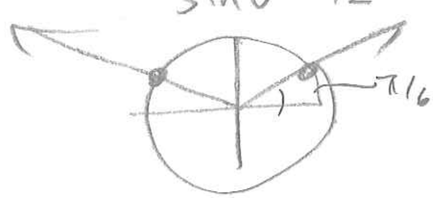
$\cancel{1} - 2\sin^2\theta + \sin\theta = 1 \quad -1$

$\sin\theta(-2\sin\theta + 1) = 0$

$\sin\theta = 0$ or $-2\sin\theta + 1 = 0$

$\sin\theta = 1/2$





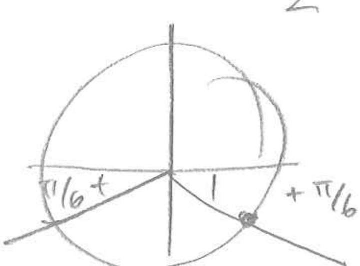
$\theta = 0, \pi, 2\pi, \pi/6, 5\pi/6$

- Find all exact solutions $0 \le \alpha \le 6\pi$

$\sin\left(\boxed{\frac{\alpha}{3}}\right) = -\frac{1}{2}$ ($\theta = \frac{\alpha}{3}$)

$0 \le \theta \le 2\pi$

$\sin\theta = -\frac{1}{2}$



$\theta = \pi + \pi/6$ Q III $= 7\pi/6$

$2\pi - \pi/6$ Q IV $= 11\pi/6$

$3\left(\frac{\alpha}{3} = \frac{7\pi}{6}, \frac{11\pi}{6}\right)$

$\alpha = \frac{7\pi}{2}, \frac{11\pi}{2}$